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Methods and Instruments for the Determination of Photographic Speeds by Measurement of Relative Characteristic Gradients*

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It has been proposed that the speed of a photographic emulsion be specified upon the basis of the exposure value at which the gradient of the H and D characteristic becomes equal to a definite fraction of the average gradient throughout the tonal range used in making the camera exposure. The adoption of such a criterion necessitates the use of easily applied methods and special instruments for the determination of gradients and gradient ratios. In this paper a number of such methods and instruments are discussed under the following headings: (1) The graphical method. (2) A device for reading speed directly from the plotted characteristic curve. (3) A direct visual photometric method (using a wedged sensitometric strip). (4) An alternative direct visual method (using a triple sensitometric wedge and a special photometric field). (5) A direct photoelectric method of measuring gradient (which reduces the error in gradient determination occurring in visual photometry). (6) A photoelectric method incorporating a constant gradient ratio device (which eliminates the need of mental calculation). (7) Making automatic the determination of speed (pointing out means for automatic recording of results). Errors introduced by non-uniformity of coating and processing and means for minimizing these errors are discussed.

NUMEROUS suggestions have been made[1] for the utilization of a definite value of gradient ($dD/d \log E$) of the characteristic curve of a photographic emulsion to define the effective speed of that emulsion. Furthermore, it appears that the preponderance of evidence[2] points definitely to the merits of a speed assignment on the basis of a limiting value of gradient, the numerical value of which is fixed by the gradient of some other portion of the characteristic curve. Jones,[3] in particular, has stated that the limiting gradient useful in the assignment of speed is influenced by what is happening throughout the whole tonal scale of the negative. Specifically, he shows that, for the case of the average pictorial subject, the ratio of this limiting gradient to the average gradient of that portion of the characteristic used in making the picture is a constant which he calls "The shadow detail compression factor." The value of this constant he found to be 0.3.

* Communication 726 from the Kodak Research Laboratories.

[1] R. Luther, "The Underexposure Period of the Characteristic Curve," Trans. Faraday Soc. **19**, 340 (1923); L. A. Jones, "The Contrast of Photographic Printing Paper," J. Frank. Inst. **202**, 469, 589 (1926); **203**, 111 (1927); **204**, 41 (1927). L. A. Jones and M. E. Russell, "The Expression of Plate Speed in Terms of Minimum Useful Gradient," Proc. 7th Intern. Congress of Photography (1928), p. 130.

[2] S. E. Sheppard, "Characteristics and Anomalies of Emulsions on Development, I," Phot. J. **50**, 190 (1926); R. Luther, "Interpretation of the Characteristic Curve," Phot. J. **49**, 185 (1925).

[3] L. A. Jones, "The Evaluation of Negative Film Speeds in Terms of Print Quality," J. Frank. Inst. **227**, 297, 497 (1939).

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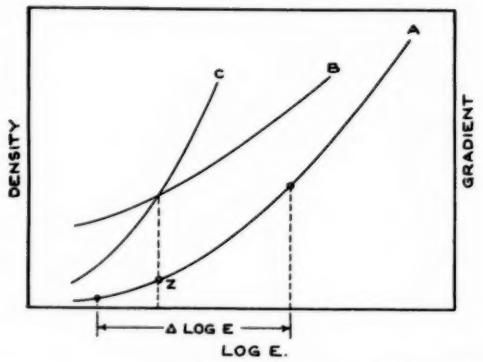

FIG. 1. Curves illustrating a graphical-mathematical method for evaluation of emulsion speed.

Photographic speed assignment upon the basis of gradient ratio measurement is already common practice in these Laboratories and it appears that a widespread adoption of similar means of speed determination in other laboratories is to be expected. It thus becomes a matter of some interest to the photographic industry to outline some of the methods and describe some of the instruments for the measurement of the speed of an emulsion as given by the $\log E$ value corresponding to the point at which the characteristic gradient is a definite fraction of the average gradient throughout a specified tonal region.

THE GRAPHICAL METHOD

The determination of the abscissa of a point on the characteristic curve at which the gradient is a specified fraction of an average slope may, of course, be made without the aid of instruments other than the standard instruments of sensitometry. Given the characteristic curve A in Fig. 1, a second curve B may be constructed, using graphically determined ordinates equal to $\Delta D/\Delta \log E$. In this case, ΔD is the density difference between points on curve A separated by a finite abscissa value $\log E$. The value of $\Delta \log E$ is customarily taken as 1.5, this being the log of the camera image brightness contrast for the statistically determined average subject. A third curve, C, is next drawn which has ordinate values equal to $n(\Delta_2 D/\Delta_2 \log E)$. The coefficient (n) is the reciprocal of the experimentally determined shadow detail compression factor (K)—numerically 3.33. The value $(\Delta_2 D/\Delta_2 \log E)$ is ideally the first derivative of the characteristic curve, but practically it may be measured graphically if the increment $\Delta_2 \log E$ is chosen small enough, say 0.2 or less.

The abscissa of the intercept of curves B and C is, of course, the value which is sought. It is the point, Z, at which the characteristic gradient is equal to $1/K$th the average gradient.

The drawing of the curves B and C is a somewhat laborious process and a very useful shortcut is provided by the semi-automatic gradient ratio-reading device which has been developed by Mr. C. N. Nelson of these Laboratories.

A DEVICE FOR READING SPEED DIRECTLY FROM THE CHARACTERISTIC CURVE

This gradient speed meter is composed of two parts as shown in Fig. 2. Part I is a rectangular transparent plastic sheet upon which are inscribed two parallel lines CD and EF. These lines are separated by a distance which is equivalent to a $\log E$ value of 1.5 on the characteristic curve which is to be analyzed. Part I also carries a small round hole at P which marks

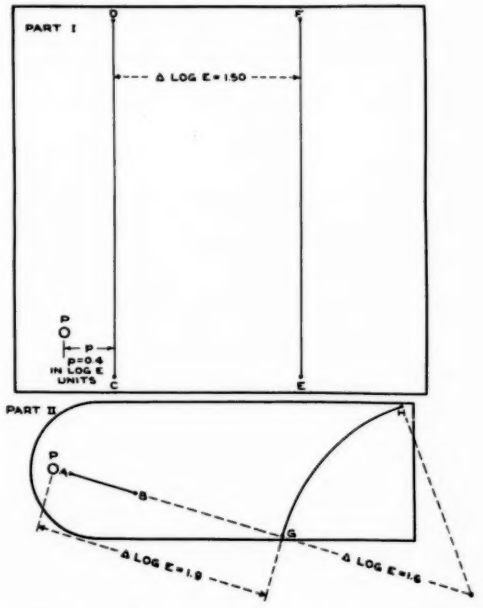

FIG. 2. Diagrammatic illustration of a transparent meter for determining speeds from the characteristic curve.

the pivot point for a transparent movable arm in Part II. P is located 0.4 log E from CD. The arm has inscribed on it the straight line AB and the curved line GH, and it is punched with the hole, P. The point G is spaced 1.9 in log E units from the point P as shown in Fig. 2. The shape and position of the curve GH on the movable arm is completely determined by the position of the pivot point P and by the requirement that the minimum gradient which is the slope of the line AB must be equal to 0.3 times average gradient. The curve GH theoretically extends to infinity. The analytical expression defining it is complicated and of no practical use. However, over its normal working range, GH is to a very high degree of accuracy the arc of a circle having a radius of 1.6 in density or log E units. When the device is assembled, Part II is fastened to Part I by a rivet through the hole P and is free to rotate about this point.

In use, the meter is placed on the H and D graph with the lines CD and EF parallel to the density axis. (See Fig. 3.) The meter is set with respect to the graph to a position at which the lines EF, GH and the H and D curve intersect at a common point while simultaneously the lines AB and CD intersect at another point on the toe of the curve. The meter parts are kept in an orientation such that these lines always intersect, while the device as a whole is slid along the characteristic curve. At some point on the curve, it will be noted that the line AB becomes tangent to the characteristic curve.

The characteristic curve at the point of tangency then has a gradient which is equal to 0.3 times the average gradient over the 1.5 log E interval. The point of tangency can usually be located after two or three trials requiring only a few seconds, and the speed of the emulsion, is, of course, quickly determined by the abscissa value of this point.

A Direct Visual Photometric Method

There is much to be said for a method of speed determination which avoids entirely the determination of the characteristic curve. If, instead of the usual step-by-step sensitometric strip, a continuous wedged exposure is impressed on the material to be tested, the gradient may

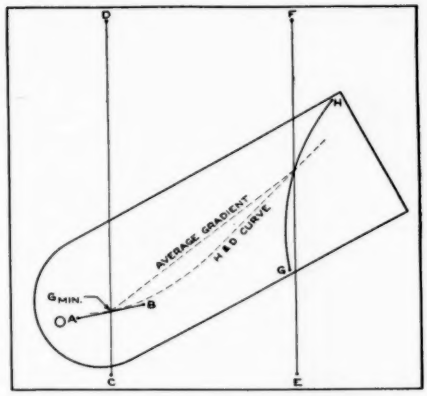

Fig. 3. The transparent meter in place on the characteristic curve.

be read directly. This type of sensitometric exposure and means for reading gradient have been described by Jones and Russell and by Jones and Tuttle.[4] Fig. 4 taken from the paper by Jones and Russell shows much a sensitometric wedge.

Any comparison beam densitometer may be used to read the gradient of a wedged strip directly, provided that it is possible to direct the beams through areas on the wedge which are separated by a known log E increment.

The conventional densitometer employed with the wedged strip will, of course, read the density difference between the two points measured. It can, however, be calibrated to read directly in gradient. In forming such a scale, the density readings are divided by the log E increment by which the beams are separated.

To construct an instrument which may be used in conjunction with wedged sensitometric impressions to determine gradient ratios, it is necessary to design an optical system or perhaps two optical systems which will pick up light from three different areas along the wedge, i.e., the second at a distance corresponding to perhaps 0.2 log exposure units from the first and the third at a distance of 1.5 log exposure units from the first.

It simplifies the optical problem somewhat if

[4] L. A. Jones and C. Tuttle, U. S. Pat. 1,762,925. (Application 1929); L. A. Jones and M. E. Russell, "Minimum Useful Gradient as a Criterion of Photographic Speed," J. Opt. Soc. Am. **25**, 401 (1935).

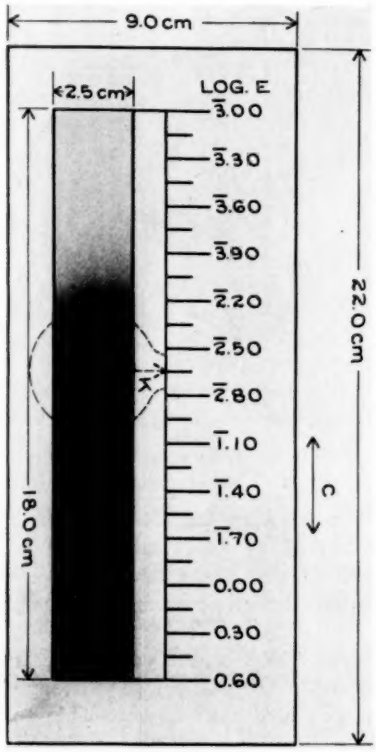

FIG. 4. Schematic plan for sensitometric strip and gradient photometer.

the sensitometric exposure is made in two halves as shown in Fig. 5. The lower half of the wedge (A) is exposed at such a level that the toe region of the emulsion is represented. The top half (B) is exposed at a level 1.5 log E steps higher than A, so that for every point on the wedge A there is an adjacent area in B which has received 32 times as much exposure. A scale S which applies to the lower wedge A gives the exposure directly in m.c.s. or in some arbitrary units. In reading, the sample is placed over an evenly illuminated opal glass* represented by the dotted circle. An opaque mask with slit areas d_1, d_2 and d_3 is placed over the opal glass. In this mask the center distances from d_1 to d_2 and from d_1 to d_3 is about 10 mm. Now, if a Martens polarization photometer is placed in contact with the sample to be measured so that the areas d_1 and d_2 are in register with the entrance pupils of the instrument, the gradient of the wedge image may be read. If the instrument is rotated 90 degrees about an axis passing through the area d_1, the comparison beam will pass through the area d_3 and the average gradient for a log E interval of 1.5 may be found.

For convenience, the angle scale of the instrument may be replaced by a scale calculated to read gradient directly, or by a scale calibrated in arbitrary numbers as explained below. The gradient scale applying to the transverse orientation of the instrument is calculated from the relation:

$$\text{Average gradient} = 0.666 \log 1/\tan^2 \theta.$$

The scale for the longitudinal orientation is calculated by:

$$dD/d \log E = 5.0 \log 1/\tan^2 \theta.$$

In practice, it is found that the portion of the scale which is used for the average gradient reading extends from 5 to 35 degrees, while the portion used in measuring the limiting gradient lies between 40 and 45 degrees. Since these portions do not overlap, both may be inscribed on a circular segment of the same radius and

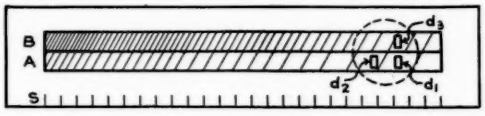

FIG. 5. Diagrammatic representation of a two-part wedged sensitometric exposure.

read with the same indicator on the analyzer of the Martens.

Figure 6 shows two different scales which are being used with this instrument. In one quadrant it will be noted that scale I reads actual values of gradient. In another quadrant scale II is marked in two series of numbers from 1 to 8. The calibration of the A and B sections of this scale is such that numbers in A represent settings at which the gradients are 0.3 times

* The diffuseness of the illumination is of very little importance in the assignment of speeds by the gradient ratio method. Even if the densitometer system is specular, so long as the Callier Q factor does not vary greatly with density, the speed reading will be the same as that obtained with a diffuse densitometer. This follows from the fact that both gradient values are factorially affected to the same extent by a change of diffuseness.

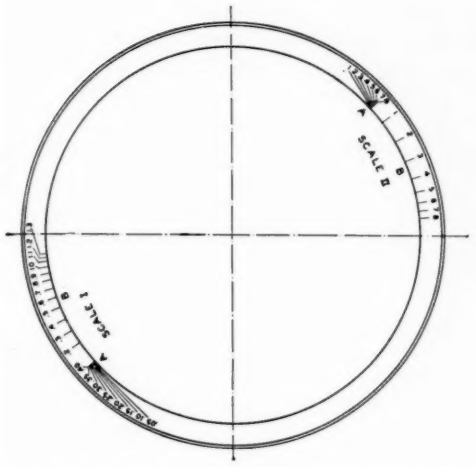

FIG. 6. Scales for a Martens photometer to read two-part wedged strips.

the gradients at the same numbers of the *B* section. Thus, having made a balanced *A* setting at a certain value, the operator is assured that if a *B* setting of the same numerical value establishes a photometric balance, speed may be read without a ratio calculation. Data for both the gradient scale and the arbitrary number scale are given in Table I. It might be noted in passing that both *B* sections are calibrated in 0.05 gradient increments. This interval was chosen because at the value of the log *E* separation appertaining to this instrument, a gradient increment of 0.05 corresponds to a density increment of 0.01 which is approximately the limit of visual photometric accuracy. A photograph of the complete instrument is shown in Fig. 7.

In using the instrument for the rapid determination of speeds, the following procedure has been found convenient:

The photometer is first set in the transverse position and balanced at some position close to the thin end of the wedged strip. The scale II number in the *A* section for this particular location is read. The Martens head is then rotated into a longitudinal position and the scale of the instrument is set in the *B* section at the same numerical value found at *A*. If the d_1 and d_2 fields happen to balance at this setting, the value of speed may be read at once from the scale printed on the wedge strip (see Fig. 5). Should the d_2 field appear brighter than the d_1 field, it will be apparent that the gradient value at the chosen point is less than that required. A few trial and error adjustments of the position of the sample strip will readily establish the position at which the limiting gradient is 0.3 average gradient.

An Alternative Direct Visual Method

A still greater simplification of the optical system needed for the assignment of speeds by the gradient ratio method is possible if the wedged strip is divided into three sections exposed so that the second wedge *B* (Fig. 8) receives at all points a small increment in exposure more than is received by corresponding

TABLE I. *Data concerning Martens photometer scales.*

Scale I Direct Gradient Scale				Scale II Arbitrary Division Scale			
Section A		Section B		Section A (Limiting Gradient)		Section B (Average Gradient)	
Scale Marking	Angle	Scale Marking	Angle	Scale Marking	Angle	Scale Marking	Angle
0.05	44°40′	0.2	35°12′	1	44°40′	1	36°48′
0.10	44°20′	0.3	30°48′	2	44°20′	2	29°36′
0.15	44°00′	0.4	26°42′	3	44°00′	3	23°00′
0.20	43°40′	0.5	22°54′	4	43°40′	4	17°. 6′
0.25	43°20′	0.6	19°30′	5	43°20′	5	13°24′
0.30	43°00′	0.7	16°36′	6	43°00′	6	10°00′
0.35	42°40′	0.8	14°00′	7	42°40′	7	7°30′
0.40	42°20′	0.9	12°00′	8	42°20′	8	5°42′
		1.0	10°00′				
		1.1	8°24′				
		1.2	7°06′				
		1.3	6°00′				

FIG. 7. Photograph of gradient photometer with Martens head.

points on A, while the third C receives 1.5 log exposure units more than does B.

With this kind of strip, the limiting gradient value may be determined by reading the density difference of a point on the first wedge and a corresponding point on the second and dividing by the log E increment by which the exposures are separated. The average gradient can be determined by a similar comparison of the second and third wedges. The scale S marked along the edge of the strip is the log E scale applying to an exposure intermediate between A and B.

A very simple photometric device is useful in analyzing such strips. Three rectangular fields are provided as shown in A, Fig. 9. These fields are of the same width as the sections of the triple wedged strip. The first field is uniform from end to end and is of the lowest brightness of the three. The second field is wedged slightly from end to end and has a slightly higher brightness at all points than adjacent points in the first field. The third field is wedged from end to end more sharply than the second field and is the brightest of the three at all points.

The triple sensitometric strip is placed upon this photometric field so that the lightest exposure is in register with field 1, while the second and third sections of the strip register, respectively, on fields 2 and 3. Now, obviously, the sample may be moved from left to right until a balance in brightness is established between fields 1 and 2 at some point along the strip. The log of the uncovered brightness difference between the two fields at this particular point divided by the log E increment between the first and second portions of the sample gives the limiting gradient.

A comparison of fields 2 and 3 may also be made. A balance is possible at the point where the log of the uncovered brightness difference between fields 2 and 3 is equal to the density difference between opposite points in portions 2 and 3 of the sample.

The average gradient is the quotient of this value over 1.5 (the log E interval). The device can be conveniently read if it is so constructed that a balance of intensity of fields 1, 2 and 3 can be obtained simultaneously straight across the three fields only when the desired gradient ratio exists.

The optical wedges which form the fields of this device may be so related to each other in gradient and absorption that the logarithmic differences in brightness between fields 3 and 2 are factorially related to the differences between fields 2 and 1. The desired numerical value of this factor depends upon the log E interval and the gradient ratio. The log E intervals between the three sensitometric wedges (Fig. 7) are 1.5

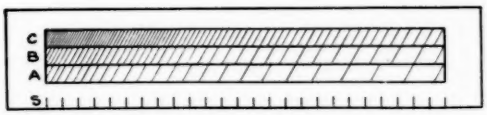

FIG. 8. Triple sensitometric wedged exposure.

and 0.15, respectively, and the desired gradient factor is 0.3. The density of the optical wedges which make up the triple photometric field to be used in analyzing such sensitometric samples may be computed from the following formula:

$$\frac{D_1-D_2}{0.15}=0.3\left(\frac{D_2-D_3}{1.5}\right),$$

where D_1, D_2 and D_3 are the densities of the

absorbing materials for any adjacent areas in a line running transversely from side to side. The wedges forming fields 2 and 3 vary in density linearly with length and may therefore be ground as straight-faced prisms from neutral density glass. Density values for the absorbing materials used in constructing the photometric field illustrated in *A* of Fig. 9 are given in Table II.

The construction of the three-component field is illustrated in *B* and *C* of Fig. 9, which shows side views of the three glass elements and an end view of the assembled unit. Number 1 consists of a block of white optical glass to which is cemented a neutral density glass (indicated in cross-hatched portion) uniform from end to end. Numbers 2 and 3 are blocks of glass having the same outside dimensions as Number 1 but composed of white optical and neutral gray glass wedges. *C* of Fig. 9 shows an end view of the assembled glass components. The top and bottom surfaces are fine ground. A uniform light source beneath this built-up cube gives uniform illumination incident on the absorbing materials and the illumination passing through the neutral absorbers is correct to measure speeds, if the average gradient lies between 0.89 and 0.33.

For accurate work it is necessary that the lines separating the photometric fields and those separating the sensitometric wedges be very sharply defined and that the strip shall register perfectly with the triple field.

In use, the triple strip is placed over the triple field and moved back and forth until some point is found which gives a uniform brightness line from side to side. The scale reading opposite this uniform line indicates the speed.

Photometric Error and Its Influence on Speed Determination

Unless a number of readings are made with the visual instruments just described, or for that matter with any other visual instrument, the determination of speed is subject to some error because of the uncertainty of photometric matching.

In practice, with the present commercial emulsions, the average gradient determinations usually lie between 0.5 and 1.0, which gives to the limiting gradients values between 0.15 and 0.33. Assuming the value of $\Delta \log E$, which is used in making the low gradient reading, to be 0.2, it is found that the values of ΔD which actually occur in the reading lie between 0.03 and 0.066. If a photometric error of 0.01 is made in the reading, the balance position at which speed is determined may be uncertain by nearly 0.2 in $\log E$ units, in the case of an

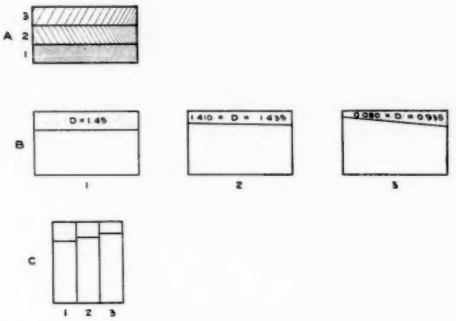

Fig. 9. Photometric device for analyzing triple sensitometric wedged exposure.

emulsion with a long sweeping toe. For the more typical emulsion, the $\log E$ error which is due to the photometric error is probably nearer 0.1. While errors of this magnitude are not particularly serious from the practical point of view, it may be desirable in some cases to increase the precision without unduly increasing the number of photometric determinations.

There is considerable evidence[5] that certain photoelectric cell arrangements are capable of an accuracy greater than that usually obtainable in visual photometry and a few of these possible applications of this nature will be briefly discussed.

A Direct Photoelectric Method of Measuring Gradients

The apparatus discussed in the succeeding section has been named by its designer, Mr. E. M. Lowry of these Laboratories, a "photoelectric ratiometer."

[5] C. Tuttle, "A Recording Physical Densitometer," J. Opt. Soc. Am. **26**, 282 (1936). C. Tuttle and M. E. Russell, "Note on the Use of an Automatic Recording Densitometer," J. Soc. Mot. Pict. Eng. **28**, 99 (1937).

The ratiometer instrument consists essentially of two crossed galvanometer coils suspended in a uniform magnetic field by means of a low torque suspension. The two coils are fastened rigidly together with their axes at right angles to each other. Such an arrangement permits the ratio between two electric currents to be readily determined. One of the currents is led through one coil and the second current is led through the other coil. When currents are flowing through the coils, each of them is acted upon by a torque which tends to bring it into a position with its axis parallel to the lines of force of the magnetic field in which it is suspended. Since the coils are fastened together, the final position of the entire movable system will be that in which the resultant of the forces acting on the coils is parallel to the magnetic vector. With equal currents flowing, the position of the coils will be the zero position. With unequal currents, the moving system will rotate and the tangent of the angle through which the coils are displaced is the ratio between the currents flowing in the coil circuits.

To apply this electrical measuring device to the problem of gradient measurements it is only necessary that the current to the coils shall be supplied by photoelectric cells which measure the light coming through the proper areas of a wedged sensitometric strip. The photoelectric cells may be of any kind but those of the barrier-layer type are more convenient to use.

The principle of the method is illustrated schematically in Fig. 10. The sensitometric wedged strip, W, is placed over three apertures which admit light from an evenly illuminated opal glass G.

Two photo-cells C_1 and C_2 measure the light transmitted by the film, W. The output from these two cells is fed into the two coils of the ratiometer. As shown in the figure, the cell C_2 is located over an aperture which is 0.15 log E units removed from C_1. It may also be moved to a position over the third aperture which is 1.5 log E units from C_1—ten times as far away as the first aperture.

TABLE II. *Densities of components of the triple field.*

	FIELD NO. 1	FIELD NO. 2	FIELD NO. 3
D min.	1.45	1.435	0.935
D max.	1.45	1.410	0.080

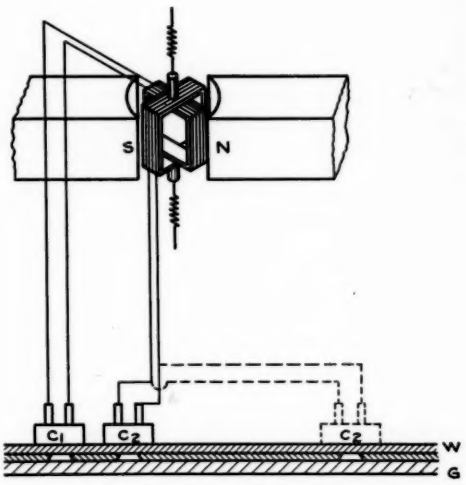

FIG. 10. The ratiometer used as a gradient meter.

The scale of the ratiometer is conveniently calibrated directly in gradient for one location of C_2 and a multiplying factor of 10 will allow the same scale to be used for the second position.

In use, the sensitometric strip is placed under the cells and the gradient is read. This reading is divided by 0.3 and C_2 is moved to the second position. The strip is shifted until by a series of approximations its correct location is found.

Experiments have proved that the probable photometric error in this application of photoelectric photometry is of the order of 0.005 in gradient.

A PHOTOELECTRIC METHOD INCORPORATING A CONSTANT GRADIENT RATIO DEVICE

The foregoing section described a method of determining one gradient first, computing the required second gradient and finding the sample setting at which this second required gradient exists. It has been demonstrated in the case of two visual instruments that neither value of the gradient need be known if the instrument is so designed that it can be read only when the gradient ratio has the correct value. The same principle applies to some photoelectric methods.

Figure 11 is a schematic representation of a photoelectric instrument incorporating this

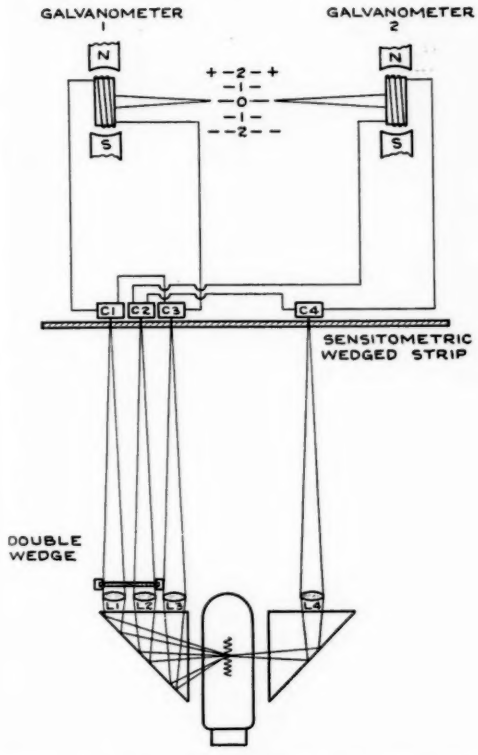

FIG. 11. A photoelectric measuring device employing a double optical wedge.

principle. In this figure, four beams of light passing through the lenses $L_1 \cdots L_4$ are incident on a wedged sensitometric strip. After traversing the strips, the beams fall on four photoelectric cells of the barrier-layer type, $C_1 \cdots C_4$. The separation of C_1 and C_3 along the strip is a log E increment of 0.2; C_2 is located between C_1 and C_3 and is separated from C_4 by a distance equal to a 1.5 log E increment.

The pairs of cells C_1–C_3 and C_2–C_4 are connected with opposed potential through the respective galvanometers Nos. 1 and 2.

A double optical wedge slides over lenses L_1 and L_2 perpendicular to the plane of the paper to control the incident light and the outputs for the cells C_1 and C_2. The density gradient of the half of the wedge which controls L_1 is made some small value, e.g., 0.1 in six inches.* The density gradient of the half of the wedge which covers L_2 is made 25 times as great as that which covers L_1.

With this double wedge entirely withdrawn from both beams, the output of C_1 will always be greater than that of C_3, if there is any gradient in the sensitometric wedged strip between these points. The same statement holds for the C_2 cell in comparison with C_4. As a consequence, both galvanometers will be deflected away from their common scale zero in the positive direction.

If the double wedge is pushed in until galvanometer 1 indicates zero, its position will be an indication of the gradient of the sensitometric strip over the log E increment C_1 to C_3. The indication of galvanometer 2 will be simultaneously zero only if the gradient over the strip for the log E increment C_2 to C_4 is equal to the C_1–C_3 gradient divided by 0.3 (multiplied by 3.33).

The simultaneous zero deflection of both galvanometers can be brought about by the correct location of the sensitometric sample relative to the four photo-cells.

MAKING THE DETERMINATION OF SPEED AUTOMATIC

Where the amount of testing which must be done justifies the additional cost involved, it will be found that a photoelectric instrument can quite simply be made automatic in its reading of the emulsion speed.

As an example, take the photoelectric gradient meter last described. The sensitometric wedged strip may be continuously driven past the four measuring stations, for instance, by placing it on a carriage which travels in gibs and moving this carriage with a motor-driven rack and pinion.

The balancing wedge should be driven by means of a reversible motor which is controlled by galvanometer 1. Perhaps the simplest means of wedge control would be the use of a double-contact galvanometer relay replacing the pointer galvanometer which is illustrated. The contact

* Since the log E increment between C_1 and C_3 is 0.2, a wedge ranging from zero density to 0.1 will measure gradients up to 0.5. Since the log E increment between C_2 and C_4 is 7.5 times as great as that for C_1 and C_3, and since it is desired to find a value of the average gradient which is 3.33 times as great as that of the $C_1 - C_3$ gradient, the density gradient of the second half of the wedge is made $7.5 \times 3.33 = 25$ times that of the first half.

points of this relay should be located one on either side of the instrument zero and so connected than an instrument deflection in either direction will make a circuit causing the wedge-driving motor to rotate in a direction which will restore the photometric balance between C_1 and C_3.

If this balance is continuously maintained as the sensitometric strip moves in the direction of increasing gradient, the point at which the proper gradient ratio occurs will be indicated by the null deflection of galvanometer 2. If this galvanometer is a relay with a contact point at its zero position, a circuit may be made which will operate a solenoid stylus to mark the speed value on the scale of the sensitometric strip.

Avoidance of the Effects of Accidental Errors in the Direct Methods of Speed Determination

It should be noted that in all the methods utilizing a wedged sensitometric strip with direct gradient reading, the accuracy with which the speed assignment is made may be influenced by small accidental variations in density at the low density end of the sample. Thus, a variation in coating thickness or evenness of development which happens to occur at the point where the final setting for the limiting gradient is made, even though the density error is small, will cause an appreciable error in the speed rating. The non-uniformity error even with careful sampling and careful processing may very likely be of the same order as the visual photometric error previously discussed and the two errors are of course additive.

A method in which the characteristic curve of the emulsion is actually drawn probably minimizes both the photometric error and the non-uniformity errors because the smooth curve represents the probable values of the location of all the points, even though the individual density readings may have some accidental error.

To eliminate the non-uniformity error, in a direct-reading method, average speed determinations of a number of samples of the same emulsion may be made but the making and reading of many samples involves so much additional labor that much of the time saved by the direct-reading method is lost.

It is possible, however, to read and plot the variation of the gradient instrument scale-reading as a function of the $\log E$ scale of the sample. From a smooth curve drawn to represent the data, the required value of speed may be determined. The actual significance of the instrument scale-reading is unimportant provided this reading is some continuous function of the gradient ratio. For example, in using the Martens gradient meter (Fig. 7), a series of gradient ratios may be read and plotted against the corresponding $\log E$ values of the sample, a smooth curve may be drawn to represent the data, and the speed may be read by interpolation on this curve at the point where the ratio is 0.3.

Similarly, the galvanometer scale readings of the double wedged instrument (Fig. 11) may be plotted against the $\log E$ values. In doing this, the sample is placed at a series of $\log E$ values, and at each setting, galvanometer No. 1 is adjusted to read zero. The readings of galvanometer No. 2 are recorded and plotted either manually or automatically. As the sample is moved to the left, the readings of galvanometer No. 2 will approach and pass through zero. The actual point at which the deflection of the galvanometer is zero may be wrong because of non-uniformity of the sample, but the speed value determined by the zero intercept of the smooth curve should be as accurate as that obtained by the more laborious graphical method involving the determination of the characteristic curve.

The non-uniformity and photometric inaccuracies occurring in the determination of the limiting gradient may also be decreased by a method whereby the low density values upon which the gradient depends are used more than once. This can be accomplished most readily by the superposition of two or more similarly processed strips in register so that the photometric balance is made with light which has traversed more than one sample. The percentage photometric error involved in reading density difference will be inversely proportional to the number of superposed strips and the probable error resulting from non-uniformity of processing is, of course, simultaneously decreased.

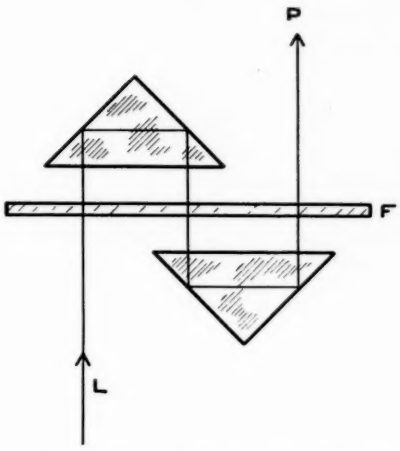

FIG. 12. Optical method of adding density.

A similar effect may be produced optically with a single sensitometric sample by any photometric system which requires that the photometric beams shall traverse the sample more than once before being received by the photometer.

The desired result can be achieved by reflecting the photometric light through two or more areas located in a line across the type of wedged exposure shown in Fig. 4. There are a great number of ways in which this may be accomplished and the one shown in Fig. 12 is intended merely to illustrate the principle. A collimated beam L is incident on one area of the film sample F. F is shown as a transverse cross section of the sensitometric wedge exposure. A totally reflecting prism redirects the transmitted light through a second area of the sample which has received an exposure identical to that of the first area. A second prism redirects the light again through the sample in the direction of one of the photometric fields, P. The second photometric field is illuminated by a similar optical system which is located along the strip at a distance $\Delta \log E$ from the first system. The numerical value of the gradient will be the quotient of the measured density difference divided by the product of $\Delta \log E$ and the number of layers through which the light has passed.

The Response of Several Fluorescent Materials to Short Wave-Length Ultraviolet Radiations

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(Received May 13, 1939)

A quartz monochromator has been used to measure the relative response of several synthetic phosphors in the region of 2200A to 3150A. The deactivating effect of copper and nickel on certain substances is represented graphically. With a vacuum grating spectrograph the response characteristics of zinc silicate and calcium tungstate have been extended to the Schumann ultraviolet region.

INTRODUCTION

LUMINESCENCE is excited in zinc sulfide and zinc silicate by ultraviolet light in the Schumann region as well as by longer wave-length ultraviolet. This was demonstrated by Rüttenauer,[1] who coated an internal and an external section of a low pressure discharge tube with luminescent material, and observed that those radiations that were absorbed by the quartz tube contributed a considerable percentage of the total visible light under certain operating conditions. Employing the quartz tube as a filter, only a rather broad spectral region is isolated, in which the efficiency of response at the various wave-lengths cannot be determined. Heyne and Pirani[2] coated a glass plate with fluorescent powders, and placed it in front of a photographic plate in a quartz spectrograph. Here only the visible light generated within the fluorescent powders by the absorption of ultraviolet radiations was recorded on the spectrogram. This method indicated the exciting wave-lengths and gave qualitative results for the various powders. That most of the light from the commercial fluorescent lumiline lamps is caused by the resonance radiation of mercury at 2537A is easily demonstrated[3] with a small quartz spectrograph.

Refinements may be introduced into the photographic method that yield quantitative data in those parts of the spectrum where it is difficult to use direct measuring devices such as a thermopile or photo-cell. This paper records an attempt to determine the relative effectiveness of the various wave-lengths in the spectral region between 1000A and 3150A to excite fluorescence in several fluorescent materials.

APPARATUS AND METHOD

Two different pieces of equipment were used to cover this spectral region. A Bausch and Lomb quartz monochromator, vacuum thermopile, photo-cells and thermionic tube amplifier furnished data for the region between 2200A and 3150A. This is a direct method, for the intensities of the spectral lines of the quartz mercury arc could be measured with the thermopile and with a calibrated thorium photo-cell. To extend the investigations into the Schumann ultraviolet region a vacuum grating spectrograph was used that gave measurable results between 1000A and 2600A. Although quantitative results were obtained with this instrument, one had to depend upon the densitometry of line images on the photographic plate for relative energy measurements, and had to assume a constant light intensity of the source and a uniform response of the photographic film over the region covered.

Figure 1 shows the arrangement of the apparatus used in determining the response of the luminescent materials to ultraviolet in the 2200A to 3150A region. The spectral energy distribution of the quartz mercury lamp is determined with the Bausch and Lomb quartz monochromator and its attached vacuum thermopile. The lamp is a standard H7 mercury lamp mounted inside an outer glass envelope. A hole was cut into the

[1] A. Rüttenauer, Zeits. f. tech. Physik **17**, 384 (1936).
[2] G. Heyne and M. Pirani, Zeits. f. tech. Physik **14**, 31–33 (1933).
[3] Marden, Beese and Meister, Trans. I. E. S. **34**, 55 (1939).

side of the glass housing to permit the use of the ultraviolet radiations. This lamp is rated at 400 watts and burns on a.c. circuits but it operated satisfactorily on d.c. at 3.2 amp. and 45 volts with the outer envelope filled with air. The resonance radiation at 2537A has an intensity comparable to that of the other strong lines under this operating condition.

The response of luminescent materials to ultraviolet light was obtained by mounting a thin microscope cover glass that had been coated with the powder in the eyepiece of the monochromator. A thin coating of nitrocellulose lacquer was brushed on the cover glass and before it was quite dry the powder was dusted on. In some cases the binder was burned off by baking at 500°C but this did not change the wave-length response curve. A caesium photocell was placed in front of the fluorescent sample to collect the visible light that was generated. A thick piece of barium flint glass was interposed between the fluorescent sample and the photocell to prevent the longer radiations near 3000A from exciting the photo-cell. With a grid leak of 4000 megohms, a floating grid circuit amplifier and sensitive galvanometer, very feeble responses were readily measured. By dividing the fluorescent response obtained with the individual ultraviolet lines by their corresponding energy values as determined with the thermopile, an efficiency curve is obtained showing the activity of the fluorescent powder to the various wave-lengths of ultraviolet radiation.

Monochromator Measurements

Data were obtained with several different kinds of willemite, three of which are recorded

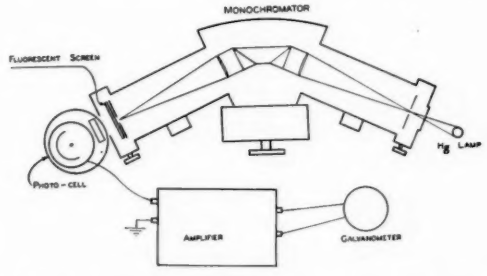

FIG. 1. Apparatus used to measure the response of fluorescent materials to radiations between 2200A and 3200A.

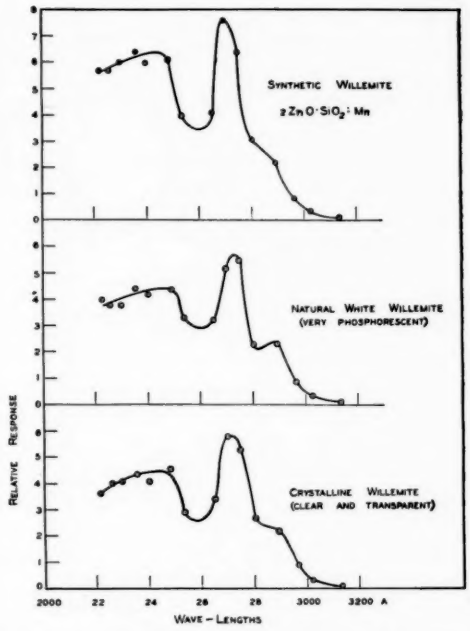

FIG. 2. Fluorescent response of several samples of willemite.

in Fig. 2. Relative responses are plotted as ordinates with wave-lengths as abscissae. All three curves show that activity increases with decrease in wave-length below 3150A, there being a pronounced maximum at about 2700A with an equally pronounced minimum at about 2600A. The curve marked "natural white willemite" phosphoresces for several minutes after it has been irradiated with short wave ultraviolet. The "crystalline willemite" was obtained at Franklin, New Jersey and was composed of several transparent crystals less than a millimeter thick and about one mm long that were ground very fine in an agate mortar. This showed no detectable afterglow. The synthetic willemite ($2ZnO \cdot SiO_2 : Mn$) made in our laboratory showed approximately the same spectral response as the natural minerals. Its phosphorescent afterglow was measured[4] in terms of 10^{-2} second. Apparently the phosphorescent property has little if any influence on the shape of the response curve in this spectral region. All samples of willemite gave off a green colored luminescence. Curves showing the spectral distribution

[4] N. C. Beese, J. Opt. Soc. Am. **29**, 26 (1939).

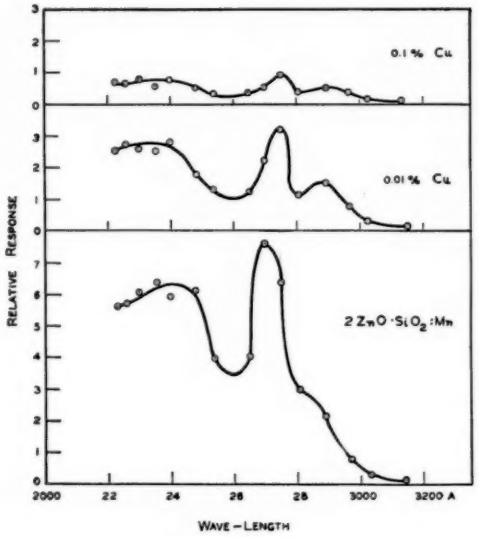

FIG. 3. Fluorescent response of zinc silicate with copper impurities.

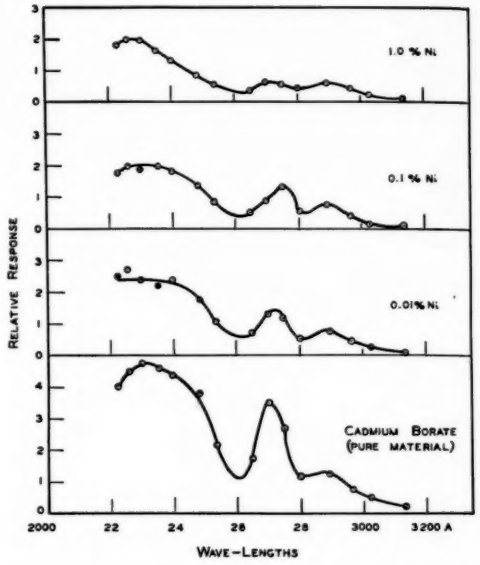

FIG. 4. Fluorescent response of cadmium borate with nickel impurities.

of the fluorescent light from willemite as well as of other luminescent materials are recorded in a paper by Inman.[5]

Synthetic powders were made in our laboratory by Dr. J. W. Marden and Mr. George Meister.[6] They supplied samples of high purity materials to which small percentages of foreign elements had been added intentionally. The effects of the added impurities are shown graphically. While copper in small percentages is an activator for some luminescent materials, notably certain sulfides, it also exerts a very poisonous effect on other substances. The deactivating property of copper on manganese activated zinc silicate is depicted in Fig. 3. Here are shown the response of the pure substance and also samples to which 0.01 percent Cu and 0.1 percent Cu have been added. After the Cu impurity was added, the mixture was fired at 1200°C for 15 minutes. The loss of activity is very apparent. The entire curve shows a decrease in light output as the copper is added to the pure material. The residual fluorescence is partially due to the response of the cover glass to ultraviolet light. The powder is not a homogeneous mass, and where radiation penetrates through the open spaces between adjacent crystals it causes slight glass fluorescence.

The poisonous effect of nickel in small percentages is shown in Fig. 4 in the deactivation

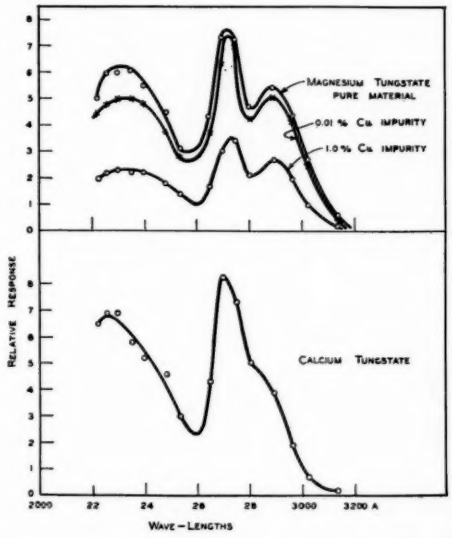

FIG. 5. Fluorescent response of calcium tungstate and magnesium tungstate with copper impurities.

[5] G. E. Inman, Trans. I. E. S. **34**, 65 (1939).
[6] J. W. Marden and G. Meister, Trans. I. E. S. **34**, 503 (1939).

of cadmium borate. In this substance there is a reduction of activity in all wave-lengths as the percentage of Ni is increased. Cadmium borate fluoresces pink.

Figure 5 shows the poisoning effect of Cu on magnesium tungstate and also compares the activity of magnesium tungstate with calcium tungstate. The activity of magnesium tungstate is considerably greater than calcium tungstate in the region between 2800A and 3150A. Magnesium tungstate fluoresces blue-white while calcium tungstate fluoresces blue.

Measurements with the Vacuum Spectrograph

Work in the Schumann region is generally confined to recording radiation intensities photographically. Our one-meter vacuum grating spectrograph followed somewhat the design of Sawyer.[7] It consisted of a main tube nine inches in diameter and four feet long that contained the grating at one end and the photographic film at the other end, with the incident light coming to the grating through a side arm set at a small angle to the axis of the main tube. The flat end plates were accurately ground and lapped so that stopcock grease applied to them produced a good vacuum seal. Provision was made to take five spectrograms each $\frac{1}{4}''$ high on the same plate, motion being imparted to the threaded screw that raised and lowered the plate holder by a pivoted lever that projected through a sylphon bellows. The movable plateholder and the grating mount were patterned after the design of Professor Shenstone's spectrograph at Princeton University. The front end plate was removed each time a new film was placed into the spectrograph. The source of light was a vacuum spark discharge from a 0.25-mf condenser charged to about 50 kv. The spark discharge occurred inside a Pyrex bulb four inches in diameter fitted with two side arms for adjustable electrodes. Pure nickel wire electrodes gave a copious supply of spectral lines of good intensity and spectral distribution. A combination of one carbon and one aluminum electrode gave fewer but more intense lines extending below 1000A. The one-meter grating was ruled on speculum metal with 6000 lines per cm.

Eastman ultraviolet sensitized film was used to record the intensities of the ultraviolet spectra. With four or five different exposures on the same film, i.e., a large variation in the number of sparks for the different spectra, a density *vs.* log exposure curve could be drawn for a number of spectral lines. The densities of the spectrum lines were determined with a microdensitometer. Exposures of 3, 10, 30, 100, 300 shots were generally used. This characteristic curve enabled one to estimate the intensities of the spectral lines that were selected for wave-length distribution. Since the intensity of the light was not constant for each individual condenser discharge—the spark breakdown did not occur at exactly the same voltage each time—the characteristic curve was not as accurately determined as with continuously operating sources of light used in the long ultraviolet or visible region. The most volatile components distilled out of the electrodes first, so the condenser was discharged a hundred or more times before light was admitted to the vacuum spectrograph. A stopcock placed in the side arm near the slit served very nicely for the shutter.

A cleared photographic film was sprayed with a thin coating of the fluorescent powders in a water suspension to form the luminescent screen. This film was mounted in the plate holder directly behind which was the active SSS panchromatic film. The incident ultraviolet radiations caused the luminescent material to give off visible light that in turn caused the panchromatic film to blacken at the location of the spectral lines. Since the film base upon which the fluorescent material was sprayed was very thin and the incident light was normal to the film, scattering of visible light was quite small, and the lines appeared quite sharp. The absorption of the gelatin film for short ultraviolet radiations provided an excellent filter to prevent the direct ultraviolet radiations from blackening the active film. Satisfactory spectrograms required about 10 times as many condenser discharges as was needed with the ultraviolet film. With a series of different exposures another photographic characteristic curve could be obtained. From this curve the intensities of the fluorescent spectra

[7] R. A. Sawyer, J. Opt. Soc. Am. **15**, 305 (1927).

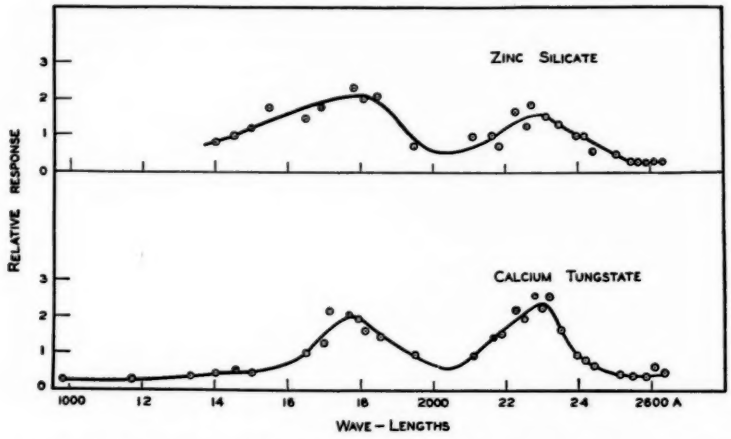

FIG. 6. Relative fluorescent response of zinc silicate and calcium tungstate in the Schumann ultraviolet region.

could be determined. Dividing the fluorescent intensities by the values of the line intensities obtained from the ultraviolet film, one obtains a relative efficiency rating or effectiveness of response for each spectrum line. Data for zinc silicate and for calcium tungstate are given in Fig. 6. The accuracy is probably not very great, but the curves show two very definite maxima, the maxima at about 2300A agree very well with the data obtained with the monochromator. This method is quite satisfactory for quantitative results below 2500A. To record fluorescent energies below 1000A very long exposures were required with our spectrograph.

CONCLUSION

Luminescence in synthetic phosphors has been activated by radiations between 1000A and 3150A and the relative response to different wave-lengths determined. The data show very decided maxima and minima. Between 2200A and 3150A spectrum line intensities were measured with direct recording instruments such as a monochromator, thermopile, photo-cell and amplifier. At shorter wave-lengths, i.e., in the Schumann ultraviolet region, a vacuum spectrograph was used where line intensities are recorded photographically. In determining intensities in the extreme ultraviolet one must assume that the response of the sensitized film to ultraviolet light of different wave-lengths is uniform. The Eastman Kodak Company has stated that "in the region down to 250A there is no gap in which the sensitivity departs from the mean level by an appreciable amount." Hence line intensities should be approximately correct. In the fluorescent spectra all lines are caused by the same color light, so the film sensitivity does not enter as an important factor.

Luminescence During Intermittent Optical Excitation

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(Received May 9, 1939)

The paper presents the results of a study of the build-up and decay of some 200 samples of silicate, tungstate, molybdate and sulphide phosphors, excited by Hg 2536A radiation. The normal silicates decay at first exponentially, with a time constant of the order of 10^{-2} sec. This initial rate is independent of the intensity and the duration of the excitation, and varies little among different samples of a given class (e.g., Zn_2SiO_4·Mn). The later decay is slower, and does vary markedly from sample to sample, and depends on the intensity and duration of the excitation. $CaWO_4$ decays too rapidly to be measured with the apparatus used (<50 microseconds); the decay of $MgWO_4$ lasts about 2×10^{-4} sec.; both these tungstates show a very faint long phosphorescence which can be enhanced slightly by addition of impurities. The red luminescence contributed by Sm in these and other "pure" phosphors decays exponentially with a time constant of about 7×10^{-4} sec. Sulphides have complicated decay characteristics, dependent on the intensity and duration of the excitation. Comparison is made with the results of a study of decay after electron bombardment, and with other published data.

I. INTRODUCTION

THIS report concerns measurements of the decay of luminescence of phosphors during intermittent excitation with ultraviolet light. The study is incomplete, but the present results allow some conclusions and some comparisons with other measurements. We hope soon to present an interpretation of part of these data, based on the zone theory of the electronic structure of solids.

II. EXPERIMENTAL METHODS

Three experimental arrangements were used.

In "Method I" the powder sample, coated thinly on a microscope cover glass, was set into a recess near the edge of a rotating disk, and was excited during part of each revolution in passing a slot illuminated with a low pressure quartz Hg arc emitting predominantly the 2536A resonance line. During the remainder of the revolution the sample ran in the dark, its decay curve being spread out as the arc of a circle. This method allowed only qualitative observations, but it was so simple that many samples could be examined in a short time (with it we have studied more than 200 specimens), it showed immediately whether color changes occur during decay, and it indicated which materials would merit more precise investigation.

In "Method II" circuits previously used for intermittent electron bombardment[1] were adapted to discharge a condenser through a low pressure quartz Hg lamp once per revolution of a variable-speed disk. The resulting brief pulse of exciting light fell on a stationary phosphor sample, and light from the decaying sample was admitted to a photo-cell through a small hole near the edge of the rotating disk. The phase between the condenser discharge and the passage of the hole across the photo-cell aperture was continuously variable over the entire cycle.

"Method III," used for most of the quantitative work, was similar to Method I: the sample, mounted near the edge of a rotating disk, was spun past a continuously-operated source lamp and then past a narrow photo-cell aperture; again the phase between beginning of excitation and measurement of the light was continuously variable over 360°.

The photo-current, in Methods II and III, was read by balancing out the voltage drop it produced in one of a calibrated set of evaporated-platinum resistors, a Compton electrometer serving as a null indicator. In all three methods the samples were excited on one side and observed from the other. The thickness was of the order of 10 mg/cm². Sometimes acetone and sometimes potassium silicate was used as a binding agent; no influence of the binder has

[1] R. B. Nelson and R. P. Johnson, Phys. Rev. **55**, 592(A) (1939); Nelson, Johnson and Nottingham, J. App. Phys. **10**, 335 (1939).

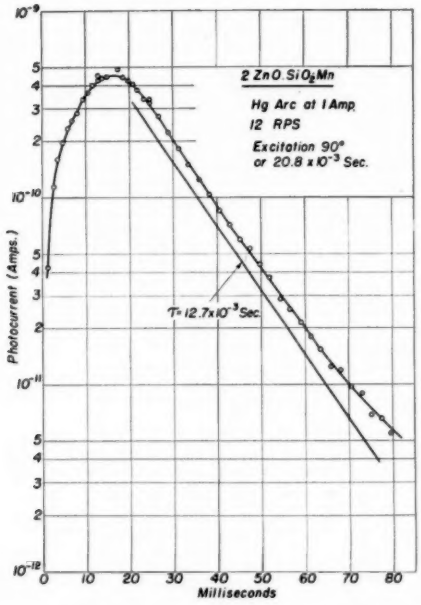

FIG. 1. Typical semi-log build-up and decay curve for a "normal" green-luminescing artificial willemite (Method III).

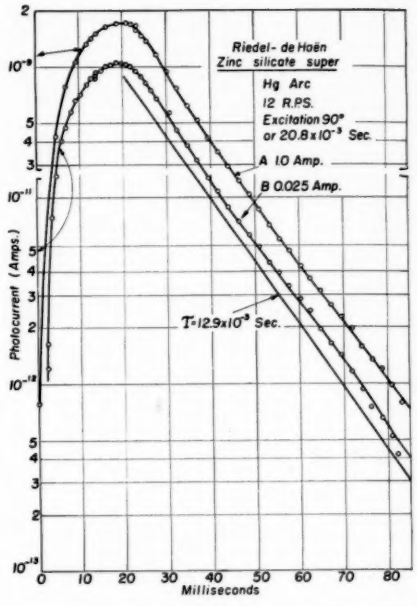

FIG. 2. Build-up and decay of a "normal" artificial willemite at different intensities of excitation (Method III). Note that the curves differ by a scale factor of about 40.

appeared in any of the tests. All the measurements were at room temperature.

III. Phosphorescence of Artificial Willemite

Many samples of green-luminescing zinc orthosilicate activated with Mn have been studied. The decay curves (Figs. 1 and 2 are typical) on semi-log paper are approximately straight, but show a slight upward concavity; the time constant is approximately 13 milliseconds. This initial radioactive type of decay has appeared, with very nearly the same time constant, for all commercial $Zn_2SiO_4 \cdot Mn$ phosphors examined. Thorough variation of the available parameters has shown that this exponential behavior, in a cycle lasting less than $\frac{1}{10}$ second, is independent of the length of the decay period, the length of the excitation period, and the intensity of the exciting radiation. These are precisely the results found after excitation of an artificial willemite (the sample of Fig. 1) by electron bombardment.[1] The conclusion is that the decay processes are identical, whether the exciting agent is electrons or Hg 2536A radiation. That the same type of initial decay also follows

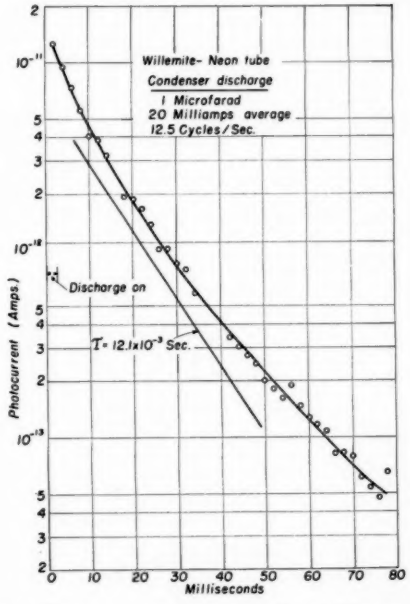

FIG. 3. Decay of a "normal" artificial willemite after brief excitation by neon resonance radiation (Method II).

excitation by the resonance radiation of neon, at 740A, is shown in Fig. 3.

None of the willemites continues to decay exponentially, at room temperature. Eventually a much slower decay sets in, and the sample can usually be seen with dark-adapted eyes for several minutes after excitation. Dr. G. R. Fonda[2] has found that at liquid-air temperature this long-lived component is frozen in, while the initial exponential decay persists with its room-temperature rate. The intensity of the long phosphorescence varies markedly from sample to sample. Fig. 4 shows decay curves for a specimen in which it was uncommonly strong. The *shape* of the curves, beyond the exponential portion, varies with the intensity of the exciting light, the other parameters being fixed.

When more Mn is present than the normal 0.1 percent to 1.0 percent, which yields the maximum efficiency of luminescence, the initial decay becomes abnormally rapid (Fig. 5), and the shape of the entire decay curve varies with the intensity of excitation. Our data on these high Mn willemites cannot be fitted over any considerable range by any empirical law of the hyperbolic form $L=a/(b+t)^{\alpha}$. With less Mn than the normal concentration, the initial decay follows the normal exponential course.

By adding a volatile flux such as $CdCl_2$ to the mixture of ingredients before firing, it is possible to form artificial willemite at a firing temperature of about 850°C, instead of having to go to about 1150°C, as in the absence of such a flux. If too little flux is present the reaction is somehow stopped short of completion and the luminescence of the resulting phosphor is yellowish instead of the normal green. We have tested two such products, finding for both the normal exponential initial decay.

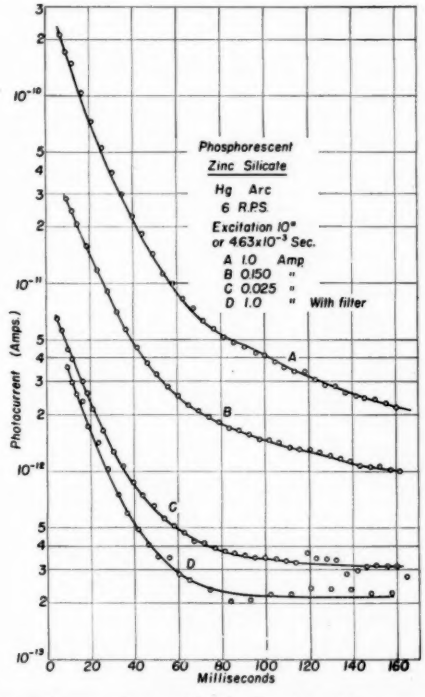

FIG. 4. Decay of an artificial willemite with pronounced persistent phosphorescence, at various intensities of excitation (Method III).

[2] G. R. Fonda, J. App. Phys. **10**, 408 (1939).

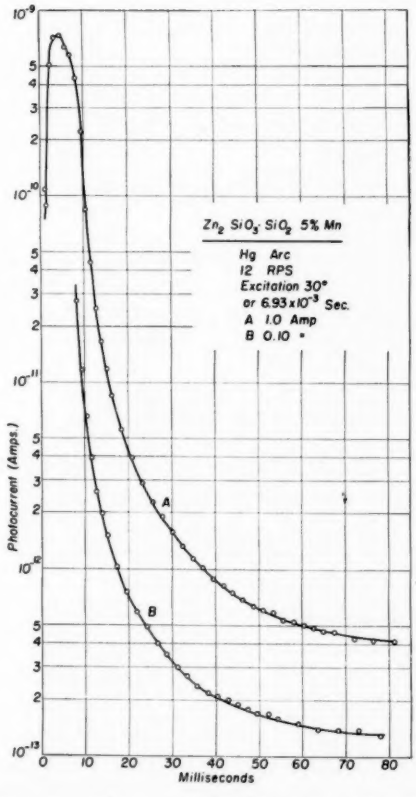

FIG. 5. Decay of an artificial willemite containing more than the optimum concentration of Mn, at different intensities of excitation (Method III).

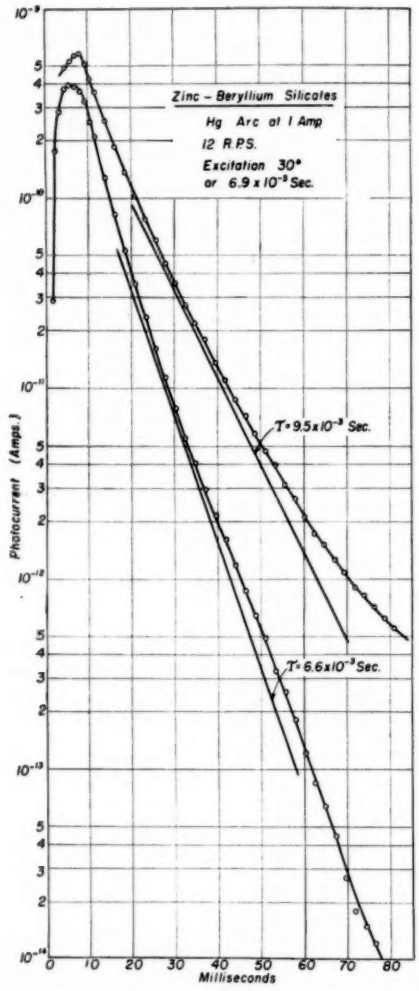

FIG. 6. Decay of two zinc-beryllium silicates (Method III). The upper curve is for a yellow-luminescing material, the lower for a red-luminescing material.

IV. Build-Up Effect in Artificial Willemite

In all the curves we have taken with artificial willemites, using Method III, the build-up portion is concave downward throughout. This is in qualitative agreement with the simplest possible hypothesis about the excitation process —that it occurs in a single stage, the rate of return of "centers" [3] from the excited state increasing monotonically with the concentration in the excited state. There is no evidence of a point of inflection, such as would appear if excitation to the emitting state were a two-stage process, with the rate of transfer from the intermediate state to the final emitting state increasing with the concentration of centers in the intermediate state.

In these experiments the intensity of the exciting light was not constant over the length of the excitation-slot, and the build-up was so rapid that very reliable values of luminescence could not easily be had, especially since large corrections for the visible light from the lamp were necessary. For these reasons no detailed study of the build-up phase was attempted. The data probably do not justify any conclusions beyond the one just proposed.

V. Phosphorescence of Zinc-Beryllium Silicates

Two samples, differing slightly in composition and in luminescent hue, have been studied with Method III. Like the green-luminescing willemites, they show an initial exponential decay

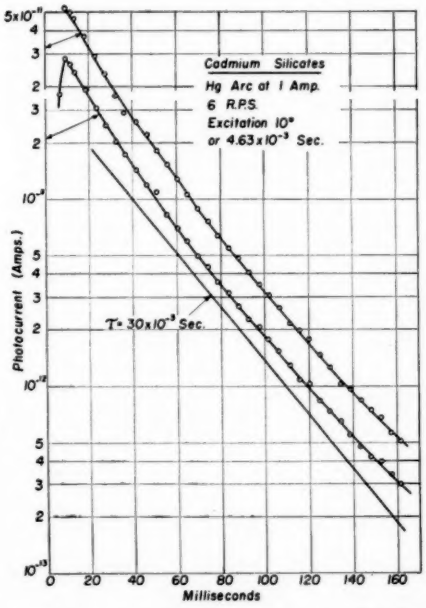

FIG. 7. Decay of two cadmium silicate samples (Method III). Note the scale-displacement.

[3] This term is used for convenience, without prejudice as to the nature of the entities involved.

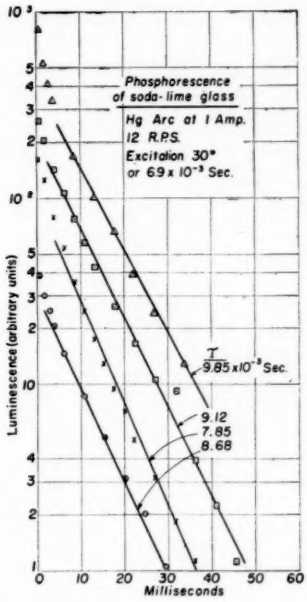

FIG. 8. Decay of yellow luminescence of four microscope cover glasses (Method III).

(Fig. 6) which is followed by a persistent weak glow. The time constant is about 9.5 milliseconds for the yellower of the two, and about 6.6 milliseconds for the redder. These data, and our observations of other zinc-beryllium silicates by Method I, are consistent with the view that addition of Be to the Mn-activated zinc silicates shifts the color of the emitted light toward the red and changes its decay rate, without making any essential change in the mechanism of decay.

VI. PHOSPHORESCENCE OF CADMIUM SILICATE·MN

Two specimens from different sources have been studied with Method III. The decay curves (Fig. 7) for the two are identical—it has been necessary to separate them by a scale-displacement in order to show them as distinct. There is an initial nearly-exponential decay with a time constant of about 30 milliseconds, followed by a slower decay too weak to appear in these curves. The color is red. The similarity with zinc silicate ·Mn, except for color and decay rate, is practically assured.

VII. PHOSPHORESCENCE OF SODA-LIME GLASS

The microscope cover glasses on which the samples were coated phosphoresce with low intensity, yielding a yellow light which decays approximately exponentially with a time constant near 8 milliseconds. Fig. 8 shows decay curves for four of these vitreous silicate samples, the light from the glass in each case reaching the photo-cell through a layer of phosphor powder.

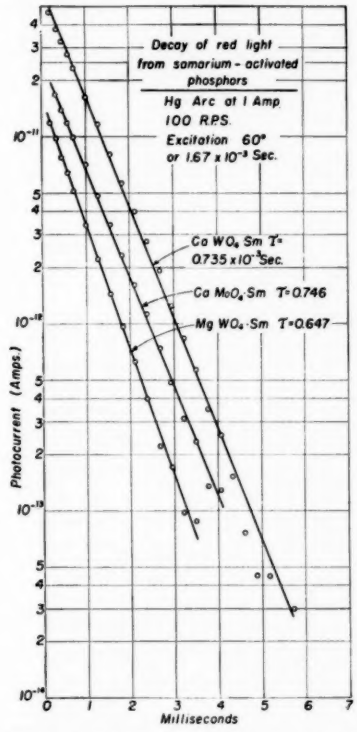
Decay of red light from samarium-activated phosphors
Hg Arc at 1 Amp.
100 R.P.S.
Excitation 60°
or 1.67 x 10⁻³ Sec.
Ca WO₄ Sm T = 0.735 x 10⁻³ Sec.
Ca MoO₄ Sm T = 0.746
Mg WO₄ Sm T = 0.647
Photocurrent (Amps.)
Milliseconds

FIG. 9. Decay of red luminescence associated with Sm in typical "pure" phosphors (Method III).

VIII. PHOSPHORESCENCE OF CALCIUM TUNGSTATE

The characteristic bright blue luminescence is extinct within 50 microseconds (the limit of observation, with Method I) after the excitation is stopped. There persists a very faint glow, barely visible, which lasts for several seconds. About 35 different elements, in concentrations of the order of one percent, have been added to this normally-pure phosphor. These all (excepting Sm —see Section X) affect only the brightness and

persistence of the long-lived phosphorescence, and none of them makes it more than barely visible. The bright blue luminescence continues to die abruptly.

IX. Phosphorescence of Magnesium Tungstate

The luminescence is blue-white. With Method I, using a very narrow excitation slot, and a disk speed of 120 r.p.s., it was possible to see that the decay is not abrupt, but lasts about 2×10^{-4} sec. after excitation. A very faint long-lived glow also appears, as in $CaWO_4$. Addition of a percent

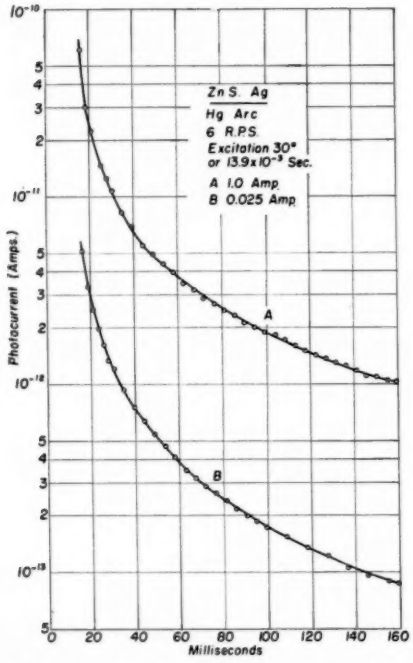

Fig. 10. Decay of ZnS·Ag, at different intensities of excitation (Method III).

or so of some 60 elements affects only the brightness and persistence of this faint component, without rendering it usefully bright, and without markedly affecting the initial rapid decay (Sm is an exception—see Section X).

X. Phosphorescence of Samarium-Activated Phosphors

When Sm is added in small concentrations to certain "pure" phosphors like $CaWO_4$ and $MgWO_4$ it contributes a red component of emitted light. This component is not easily visible except with a spectroscope, during the excitation, since it is weak in comparison with the luminescence of the matrix phosphor. In decay, however, it stands out brilliantly after the light characteristic of the unadulterated material has rapidly died away. We have measured, using Method III, decay curves for this red phosphorescence, associated with Sm, in three different matrices. The decay (Fig. 9) is exponential, and neither the rate nor the brightness differ markedly among these three samples (nor among the various other Sm-activated phosphors, including $CdWO_4$ and Zn_2WO_5, which we have studied by Method I). It is interesting that this red component does not appear when Sm is added to $MgMoO_4$, which is not itself an efficient phosphor.

XI. Phosphorescence of Sulphides

In the experiments with electron bombardment[1] it appeared that the two sulphide samples studied were quite unlike the willemite sample in decay characteristics. The decay was not exponential, and the *shape* of the decay curves varied with the density of the electron current. At the higher current densities the initial decay was more rapid, and the "final" brightness toward which the decaying sample leveled off was

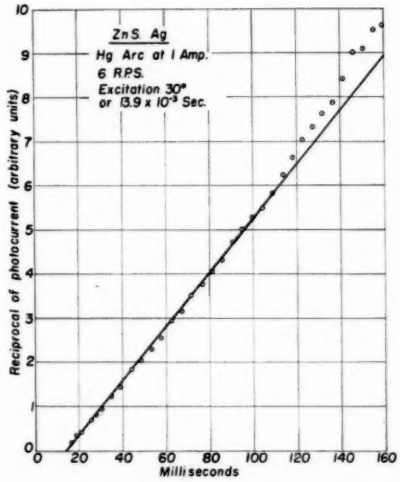

Fig. 11. The data of Fig. 10(A), plotted to show agreement with the empirical decay law $L=a/(b+t)$.

higher. We have tested these same two phosphors with excitation by Hg 2536A radiation. Fig. 10 shows decay characteristics for RCA FM-266 (ZnS·Ag, luminescing blue) at two values of current through the exciting arc (Method III). The difference in shape here is not so marked as in the electron-bombardment curves, where a much wider range of intensity was available, but is still quite definite. Attempt to fit the data of Fig. 10(A) by an empirical law leads to the expression $L=a/(b+t)$. Fig. 11 shows that this law is obeyed quite well during the first 80 milliseconds of decay. The data on decay after electron bombardment were fitted by this same hyperbolic law. Fig. 12 shows two decay curves for RCA FM-267 (ZnS·CdS·Ag, luminescing yellow) at different arc currents. In Fig. 13 the data of Fig. 12(A) are shown to fit the empirical law

$$L=c/(d+t)^2,$$

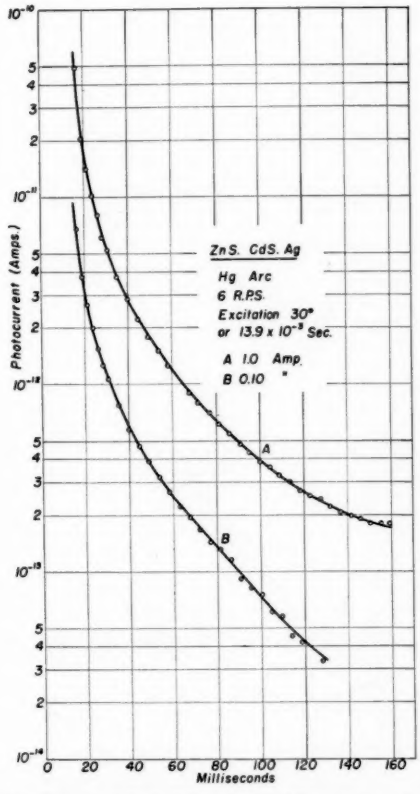

FIG. 12. Decay of ZnS·CdS·Ag, at different intensities of excitation (Method III).

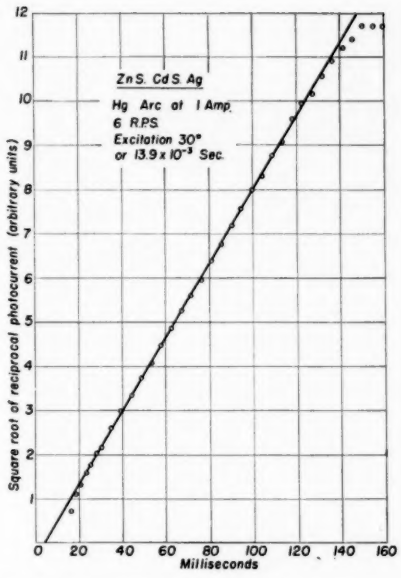

FIG. 13. The data of Fig. 12(A), plotted to show agreement with the empirical decay law $L=c/(d+t)^2$.

which happens to be the equation describing the rate of a bimolecular gaseous reaction. Again, this same type of law was found to fit the decay of this material after electron bombardment.

This ZnS·Ag sample shows a very marked persistent luminescence, lasting visibly for about ½ hour. The ZnS·CdS·Ag sample, on the other hand, decays out of sight in about 10 sec. after excitation is stopped.

We have examined some two dozen sulphide samples by Method I, finding a wide variety of behavior. It is generally true that the decay depends on the intensity of the exciting radiation, and on the length of the excitation period, as could be seen by interposing filters or stops which affected only one-half of the spinning sample.

XII. SUMMARY

Table I lists the salient results of this study. It is to be understood that the specifications (such as "exponential decay") are all subject to the exceptions noted in the body of this report, and are at best only approximately valid, the degree of their validity varying from sample to sample. On the other hand, our experience with the many samples studied qualitatively with

TABLE I.

PHOSPHOR	LUMINESCENT COLOR	EXCITING AGENT	DECAY LAW	TIME CONSTANT (SEC.)
$2\,ZnO\cdot SiO_2\cdot Mn$ (Willemite)	Green	Electron Beam	~Exponential	$\sim 13\times 10^{-3}$
$2\,ZnO\cdot SiO_2\cdot Mn$ (Willemite)	Green	Ne 740A	~Exponential	$\sim 13\times 10^{-3}$
$2\,ZnO\cdot SiO_2\cdot Mn$ (Willemite)	Green	Hg 2537A	~Exponential	$\sim 13\times 10^{-3}$
Zinc-beryllium silicate	Yellow	Hg 2537A	~Exponential	$\sim 9.5\times 10^{-3}$
Zinc-beryllium silicate	Pink	Hg 2537A	~Exponential	$\sim 6.6\times 10^{-3}$
Cadmium silicate	Pink	Hg 2537A	~Exponential	$\sim 30\times 10^{-3}$
Soda-lime glass	Yellow	Hg 2537A	~Exponential	$\sim 7\times 10^{-3}$
$ZnS\cdot Ag$	Blue	Electron Beam	$L=\frac{a}{b+t}$	
$ZnS\cdot Ag$	Blue	Hg 2537A	$L=\frac{a}{b+t}$	
$ZnS\cdot CdS\cdot Ag$	Yellow	Electron Beam	$L=\frac{c}{(d+t)^2}$	
$ZnS\cdot CdS\cdot Ag$	Yellow	Hg 2537A	$L=\frac{c}{(d+t)^2}$	
$CaWO_4\cdot Sm$	Red in decay	Hg 2537A	~Exponential	$\sim 7.5\times 10^{-4}$
$CaMoO_4\cdot Sm$	Red in decay	Hg 2537A	~Exponential	$\sim 7.5\times 10^{-4}$
$MgWO_4\cdot Sm$	Red in decay	Hg 2537A	~Exponential	$\sim 6.5\times 10^{-4}$

Method I gives us confidence that the general behavior specified is characteristic of the *classes* of phosphors, and not merely of the particular samples we have examined in detail.

XIII. COMPARISON WITH THE RESULTS OF OTHERS

We concern ourselves here only with very recent literature on the decay of phosphors.[4] Fonda's results with willemites,[2] with excitation by Hg 2536A, are generally confirmed in the present study. Schleede and Bartels[5] have recently taken cyclic excitation-decay curves for willemite and for ZnS, using a photo-cell, amplifier, and cathode-ray oscillograph. Their results, necessarily qualitative, also agree with those presented here. In particular, their curves show that the initial decay rate of sulphides is much more strongly dependent on degree of excitation than is that of willemite. De Groot's oscillographic curves[6] on sulphide samples, at various temperatures, various wave-lengths of exciting light, and various intensities, agree with our results qualitatively, and are consistent with the extensive literature in showing that the decay of sulphides is a very complicated affair. Beese[7] seems to be unique in his finding that the particular sample of green-luminescing zinc silicate he studied had a precipitate decay (within 10^{-5} sec.) down to about $\frac{1}{3}$ of its initial intensity, and thereafter decayed according to a type of hyperbolic law. This initial abrupt decay appeared only after the original data, which did not show it, had been corrected to correspond to a square wave of exciting radiation. We suggest that such correction requires considerable circumspection, and is in principle impossible unless an *a priori* choice of a decay law has been made.

We are indebted to Dr. G. R. Fonda for furnishing us with most of the samples used in this study, and for several helpful discussions.

[4] It appears that the older phosphors differed chiefly in efficiency, not in any essentials of behavior, from the materials recently developed for television and fluorescent lighting. See, for example, E. L. Nichols and E. Merritt, *Studies in Luminescence* (Washington, 1912). A fairly comprehensive bibliography is given by P. Pringsheim, *Fluorescenz und Phosphorescenz* (Berlin, 1928, third edition); for a supplementary bibliography see H. Rupp, *Die Leuchtmassen und ihre Verwendung* (Berlin, 1937).

[5] A. Schleede and B. Bartels, Zeits. f. tech. Physik **19**, 363 (1938).

[6] W. de Groot, Physica **6**, 275 (1939).

[7] N. C. Beese, J. Opt. Soc. Am. **29**, 26 (1939).

The Dispersion and Transmission of Methyl Methacrylate Polymer*

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(Received May 16, 1939)

METHYL methacrylate monomer, an organic liquid of definite chemical constitution is transformed into a solid (polymer) by the addition of a suitable catalyst or upon exposure to light and heat. Because of its remarkable clarity and because of its possible use as an optical medium, this polymer was studied for the purpose of determining its dispersion and transmission in the ultraviolet, visible, and near infra-red regions of the spectrum.

Dispersion measurements were made with a prism whose faces were 65×65 mm and whose angle was approximately 30°. The water-white polymer was prepared by the E. I. duPont de Nemours Company, and the prism was ground and polished in the optical shop of this University.† Heretofore the determination of the dispersion over so wide a spectral range not only called for much accessory apparatus but involved much labor. In carrying out the present work it was found that, through the use of photography, the operation could be simplified greatly with no sacrifice in accuracy. By means of a spectrometer supplied with a divided circle the prism angle A and the angle of minimum deviation D for the 5461 mercury line were determined visually. From these measurements the refractive index n_{5461} was evaluated in the customary manner. The prism was then mounted so as to form part of the mirror-spectrograph shown in Fig. 1. Here, the radiations from the quartz mercury arc A proceed through the slit B to the concave speculum mirror C which sends parallel rays through the polymer prism D to the second, similar, concave speculum mirror E (97.8 cm focal length) which, finally, projects the resultant spectrum on a photographic plate F. In addition to the customary adjustments which must be made in setting up a spectrograph of this type, it is essential that the prism again be adjusted to minimum deviation for the 5461 mercury line. Clearly, all other radiations from the mercury arc will not pass through the prism at minimum deviation—as for example the radiation ($\lambda=3650$A) which follows the course shown in Fig. 2. For this (3650) radiation, we have the relation

$$\frac{\sin i_1}{\sin r_1}=\frac{\sin i_2}{\sin r_2} \qquad \sin r_2=\frac{\sin i_2 \sin r_1}{\sin i_1}. \tag{1}$$

Again $r_1+r_2=A$, hence $r_2=A-r_1$. Expanding:

$$\sin r_2=\sin A \cos r_1-\cos A \sin r_1. \tag{2}$$

Equating (1) and (2)

$$\sin i_1 \sin A \cos r_1-\sin i_1 \cos A \sin r_1 = \sin i_2 \sin r_1$$

$$\frac{\sin r_1}{\cos r_1}=\tan r_1=\frac{\sin i_1 \sin A}{\sin i_1 \cos A+\sin i_2},^{**}$$

$$r_1=\tan^{-1}\frac{\sin i_1 \sin A}{\sin i_1 \cos A+\sin i_2}, \tag{3}$$

whence, finally

$$n_{3650}=\sin i_1/\sin r_1.$$

All quantities on the right side of expression (3) are known: i_1 (the same for all wave-lengths) is calculated from the known values of A, D and n for the 5461 line; i_2 is known since, on the emergent side of the prism, $i_2=i_1$ for the 5461 line. For other lines $i_2=i_1\pm\Delta i$ where Δi may be evaluated from the known focal length of the concave mirror E and from the linear separations of the various spectrum lines from the 5461 line.

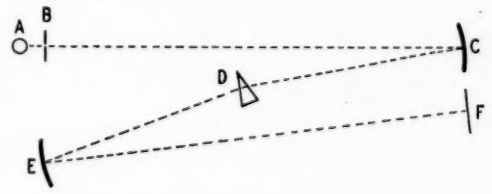

FIG. 1. Design of the mirror spectrograph used in this work.

* This material is known, commercially, as "Lucite."

† I am indebted to Mr. W. H. Perry for carrying out this operation.

** It is more than probable that this simple proof has been given before. The mere fact that it is not well known is sufficient excuse for this presentation.

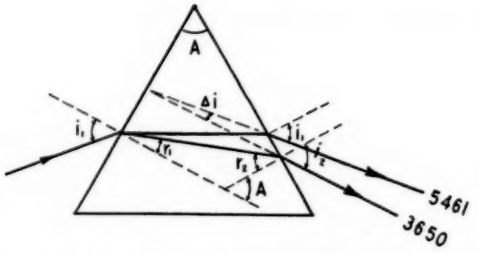

FIG. 2. Tracing the course of a ray (5461) which passes through the prism at minimum deviation and of another ray (3650) not passing at minimum deviation.

This means, then, that from a single spectrogram it is possible to evaluate the dispersion of the material at hand from its limit of transmission in the ultraviolet near (3130A) to the limit of sensitivity of the photographic plate around 10,140A in the near infra-red.* As a matter of fact, it is quite feasible to adapt Czerny's[1] method of "evaporography" to this problem and thus to extend dispersion measurements out to about 10μ.

For photography of the spectrum, Eastman infra-red plates, sensitized with ammonia were used. An exposure of two hours was given with a piece of Corning signal-red glass in front of the slit; this exposure was followed by one of five minutes with a yellow filter so as to bring out the green and yellow lines; finally, an exposure of 30 seconds without any filter was given. The resultant spectrogram showed 11 sharp, well-spaced lines extending from 3130 to 10,140A.

TABLE I.

λ	Δl	Δi	i_2	r_1	n
3130A	1.835 cm	1.075°	23° 46.0′	14° 39.3′	1.5247
3341	1.463	0.857	33.0	43.3	1.5180
3650	1.060	0.621	18.8	47.7	1.5108
4047	0.689	0.404	5.8	51.7	1.5041
4358	0.471	0.276	22° 58.1	54.2	1.5000
4916	0.190	0.111	48.2	57.3	1.4948
5461	0	0	41.5	59.5	1.4914
5780	0.085	0.049	38.6	15° 0.4	1.4898
6907	0.305	0.179	30.8	2.9	1.4859
7729	0.411	0.241	27.1	4.1	1.4840
10140	0.578	0.339	22° 21.2	6.0	1.4809

* Before beginning to make measurements the prism was viewed between crossed Nicols which revealed slight double refraction. All of the prism faces but central areas 15 mm in diameter was blocked off with cardboard, since this area appeared black between the crossed Nicols.

[1] M. Czerny, Zeits. f. Physik **53**, 1–12 (1929); M. Czerny and P. Mollet, Zeits. f. Physik **108**, 85–100 (1937).

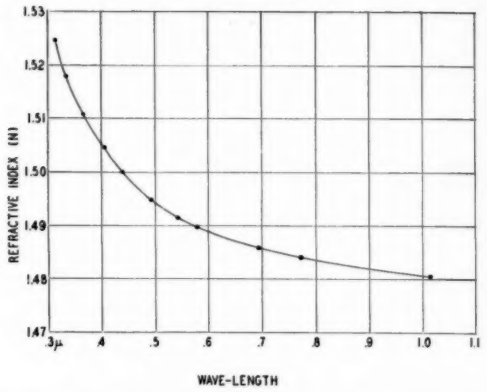

FIG. 3. The dispersion curve of methyl methacrylate polymer at 23° C.

The constants and symbols used in the following evaluations are: A (prism angle) = 29° 58.9′, D (minimum deviation $\lambda = 5461$) = 15° 24.2′, $n_{5461} = 1.4914$ at 23°C, i_1 (fixed angle of incidence) = 22° 21.5′, i_2 (angle of emergence) = $i_1 \pm \Delta i$, Δl = linear separation of lines on spectrogram from the 5461 line, F = focal length of concave mirror $E = 97.8$ cm, $\Delta i = \Delta l / F$. The results are given in Table I.

These results, plotted in Fig. 3, are presented with considerable confidence in their relative accuracies. By means of a dividing engine the positions of the lines on the negative were determined with a maximum error of setting of 0.002 cm which introduced an uncertainty of 0.000016 in the value of n, an uncertainty quite beyond the reliability of our spectrometer. A further check was realized by determining also the value of the refractive index for the 4047 line with the spectrometer and by comparing this value with the one obtained from the spectrogram. The difference between the two values was 0.0002. Because of imperfections in the planarity of the prism faces and in the spectrometer itself, the absolute values of the refractive index are considered reliable to within ±0.00025 while the relative values obtained by photography are of the greater accuracy mentioned previously. It is felt that there is no point in improving the absolute accuracy of these results, since the value of n_D for various polymers of this type may differ by as much as 0.002.

In general, the dispersion $(n_F - n_C)$ of the

polymer is of the order of magnitude of that characteristic of crown glass, whereas the value of n_D is distinctly less. These facts become evident from the analysis of the numerical values presented in Table II.

Transmission measurements in the visible and near infra-red regions of the spectrum were made with a glass monochromator and thermopile, while measurements in the ultraviolet were carried out with a quartz (fluorescence) spectrophotometer. The sample was a well-polished disk of methyl methacrylate 5 mm thick. The results are shown in Fig. 4.

In comparison with plate glass of the same thickness, the polymer is considerably more transparent in the ultraviolet and more opaque in the infra-red. Plate glass is already opaque at 3200A, whereas the polymer transmits almost down to 2900A. A polymer film 0.12 mm thick was found to transmit even beyond 2536A. The increasing opacity in the infra-red is due to absorption bands near 1.16, 1.40, 1.72μ and beyond. These absorption bands are characteristic of definite linkages in the organic molecule.

TABLE II.

MEDIUM	n_D	$n_F - n_C$	$\frac{n_D - 1}{n_F - n_C}$
Ordinary crown glass*	1.5171	0.00868	59.6
Light barium flint glass	1.5682	0.01164	48.8
Methyl methacrylate polymer	1.4893	0.0085	57.6

* *Handbook of Chemistry and Physics* (Eighteenth edition, 1933), p. 1404.

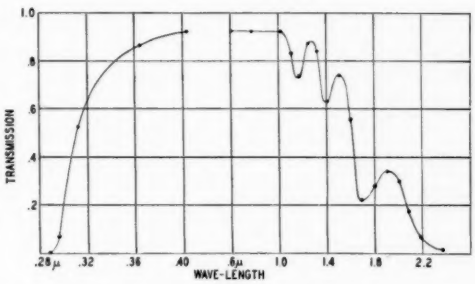

FIG. 4. The transmission curve of a methyl methacrylate polymer plate 5 mm thick.

The writer acknowledges with pleasure his indebtedness to Mr. W. G. Fastie for substantial help in carrying through this investigation.

Derivation and Significance of the So-Called "Chronotopic Interval"

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(Received May 10, 1939)

Starting with the Fitzgerald-Larmor-Lorentz contractions of length and frequency the measurements which can be made on a uniformly moving particle from a uniformly moving platform, using clocks and rods moved at observed finite speeds, are determined. A single quantity in the Fitzgerald-Larmor-Lorentz invariant framework is found to be determinable from such measurements. This, the so-called "chronotopic interval" is a length, which is identified with a stationary undistorted dimension of an interferometer, which, experiencing the length and frequency contractions on motion, always gives a "null" result. The complete expression for this quantity in terms of the variant rods and clocks is given by a formula which is not identical with the invariant framework expression, but is a function of all the observed quantities, including the observed velocities of the rods and clocks. Applying the same analysis to observations made in a gravitational field, on certain assumptions, an expression is found for the "chronotopic interval" or interferometer dimension, which is a function of the gravitational constant when expressed in the invariant framework, but is identical with the non-gravitational formula when expressed in terms of variant rod and clock measurements.

INTRODUCTION

THE recently described observation of the unsymmetrical Doppler effect in hydrogen canal rays[1] constitutes the necessary link to confirm the theory of Fitzgerald, Larmor and Lorentz on the behavior of matter in motion, a theory developed to explain the *null* result of the Michelson-Morley experiment. In this theory matter is considered as moving through a fixed framework designated as the "luminiferous ether." To be exact the experimentally observed framework is the coherent pattern of radiant energy reaching us from the fixed stars, of which our locally generated light signals become a part. Motion relative to this radiant energy pattern is revealed to us by a series of optical experiments involving rotation, namely Bradley's aberration of light, occasioned by the annual orbital motion of the earth, the Michelson-Gale experiment showing the diurnal rotation of the earth, the Sagnac experiment,[2] which may be performed with a revolving platform in the laboratory. Only when the radius of curvature of the motion becomes infinite does the relative motion of matter and the radiant energy pattern escape observation, and it is to explain this limiting case that Fitzgerald, Larmor and Lorentz postulated their contractions of length and clock-rate, —the latter being the phenomenon recently demonstrated experimentally as a *positive* effect.

As a necessary preliminary to the problem to be discussed in this paper the characteristics of this radiant energy pattern, hereafter referred to as the *invariant framework*, and of the measuring instruments to be used in the study must be precisely defined. This is done in the following three postulates.

(1) Velocities of particles moving in the invariant framework, are determined by the ratio of the time taken by a moving particle to traverse the distance between two points at rest in the framework, as measured by rods and clocks which are unaffected by motion in the framework (referred to hereinafter as invariant rods and clocks).

(2) The velocity of light in the invariant framework, measured as in (1) is a constant c.

(3) The lengths of material rods (in the direction of motion) and the frequencies of material clocks (hereinafter referred to as variant rods and clocks) are reduced, when in uniform motion, through the invariant framework, in the ratio $(1-v^2/c^2)^{\frac{1}{2}} : 1$, where v is the velocity of the rod or clock, and c is the velocity of light, both velocities being measured as in (1).

This invariant framework is essentially identical with the luminiferous ether of Fitzgerald, Larmor and Lorentz. The requirement that the rods and clocks used to establish distances and times in the invariant system *shall be unaffected*

[1] H. E. Ives and G. R. Stilwell, "Experimental Study of the Rate of a Moving Atomic Clock," J. Opt. Soc. Am. **28**, 215 (1938).
[2] H. E. Ives, "Light Signals Sent Around a Closed Path," J. Opt. Soc. Am. **28**, 296 (1938).

by motion is actually equivalent to Lorentz' use of "stationary" rods and clocks, but is more completely descriptive and useful,—Lorentz' "stationary" rods and clocks are rods and clocks which, even when moved, give readings *as though stationary*. The assumption of such invariant rods and clocks is implicit in all discussions of this subject before the time of Fitzgerald and did not need explicit statement since the idea of variant rods and clocks had not been suggested. Whether invariant rods and clocks exist physically is immaterial for the argument. They are just as legitimate tools for theoretical study as pendulums of invariant length before physical means were found for compensating for temperature variations, or as various other devices such as ideal heat engines and perfectly reflecting enclosures, which have rendered a valuable service in theoretical physics.

It is the purpose of this paper to discuss the measurements of the uniform motion of a particle which can be made with rods and clocks of the type postulated in (3). It is obvious that measurements made with such rods and clocks will be functions of the speeds with which the rods and clocks are manipulated, and hence will differ from measurements made in the completely invariant system. The special case of light signals (velocity of moving particle $=c$) has been previously discussed.[3] It was developed in that discussion that the several factors entering in, namely the velocity of the observing platform, and the distances and times, as they would be measured by *invariant* rods and clocks, are indeterminate because of the rod and clock rate contractions. Only one quantity of the invariant system, namely *the velocity of light*, is determinable as a function of the observable quantities. In the present discussion we shall investigate whether with a particle moving at speeds less than the velocity of light, *any* invariant quantity can be determined. The invariant quantity which we shall find, which has been variously called the "line element," the "interval," the "chronotopic interval" will be interpreted and its significance discussed.

In the present discussion the problem will be treated with one simplification, as compared with the previous study of light signals as measured with transported rods and clocks. Here we shall consider only the effect of moving our clocks at finite speeds. Distances will be measured by measuring rods moved at zero speed with respect to the observation platform; that is, a "standard" rod will be laid alongside the measuring platform, or distances will be measured off by laying a rod or rods end to end in successive stationary positions on the measuring platform, instead of observing a rod in motion past the platform. This simplification is possible without damage to the essential character of the result, because, as shown by the previous work,[3] the consequence of using a moving measuring rod is to introduce a simple multiplying factor, constant for any given observed rod velocity, into the values of the length measurements. The case is quite different with the use of transported clocks, where a change in the form of the expressions is brought about. This difference in the consequences of using transported rods and of using transported clocks arises from the fact that there is nothing cumulative about the contraction of length produced by motion—on coming to rest with respect to a moving platform a rod resumes the value normal to the velocity of the platform. On the other hand, a clock whose rate has altered during its manipulation accumulates *a difference of setting* which is not annulled by resumption of a former rate.

Distances and Times Measured by Transported Clocks

We now proceed to develop the expressions for the distance and time of travel of a particle moving uniformly between two fixed points a and b with the velocity $u\{=(x^2+y^2+z^2)^{\frac{1}{2}}/t\}$ as measured on a physical observation platform moving with uniform velocity v (as measured in the invariant framework) in the positive X direction. We confine ourselves entirely to the use of transported rods and clocks. The treatment thus differs from that used by Einstein in developing his Special Theory of Relativity, in which, while rods are used as we shall here, the only clock is one stationary on the moving observation platform, and time at another point is fixed, by his *definition* of simultaneity, as being $\frac{1}{2}$ the time,

[3] H. E. Ives, "Light Signals on Moving Bodies as Measured by Transported Rods and Clocks," J. Opt. Soc. Am. **27**, 263 (1937).

as measured by this clock, for *a light signal* to go to the other point and back. The time so defined, as previously developed,[3] is that given by the special case of transporting the clock infinitely slowly (with respect to the moving platform) to the distant point.

We now consider first the distance traversed by the moving particle, as measured in the moving system. We have for the X component of the distance on the moving platform, the distance

$$x-vt,$$

where x is the distance moved in the invariant framework and t is the time of travel in the same system. Owing to the contraction of the moving observation platform in motion this distance, x', measures as

$$x'=\frac{x-vt}{(1-v^2/c^2)^{\frac{1}{2}}}. \tag{1}$$

For the Y and Z components we have, because of the absence of contraction, the simple relations

$$\begin{aligned} y'&=y,\\ z'&=z. \end{aligned} \tag{2}$$

We next take up the time of travel of the particle, as indicated by a clock on the moving platform, which, set to agree with a clock at a at the beginning of the particle's travel, has been moved at finite velocity to the point on the moving platform at which the particle finally arrives. For simplicity of treatment we shall move the clock by steps, first along the X axis, for the distance $x-vt$, then along the Y and Z axes of the moving platform. With respect to the moving body let the velocity of the clock (in invariant units) be w; its velocities in the three directions it is to be moved are then (in invariant units), $w+v$, $(w^2+v^2)^{\frac{1}{2}}$, $(w^2+v^2)^{\frac{1}{2}}$. Considering first the movement in the X direction, we have that the invariant time (T) of this movement is given by

$$(v+w)T=(x-vt)+vT,$$

from which

$$T=\frac{x-vt}{w}. \tag{3}$$

Now this time as indicated by the moved clock will be

$$T'=T\left(1-\frac{(v+w)^2}{c^2}\right)^{\frac{1}{2}}=\frac{x-vt}{w}\left(1-\frac{(v+w)^2}{c^2}\right)^{\frac{1}{2}}. \tag{4}$$

After it has come to rest on the moving body this clock will be set back by the difference in indication between it and the clock which remained at the origin, which indicates the time as $T(1-v^2/c^2)^{\frac{1}{2}}$, by

$$\Delta S=\frac{x-vt}{w}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{(v+w)^2}{c^2}\right)^{\frac{1}{2}}\right]. \tag{5}$$

Now w has been expressed in invariant rod and clock units, which are, because of the contractions of physical rods and clocks, unobservable; let us express it as an *observable* velocity. For this purpose we watch the number of divisions on the moving platform which pass the clock per unit of time, as measured by the clock. An observer on the clock observes the velocity w increased inversely as the length of the divisions, and inversely as the clock rate, or, designating the observed velocity as w_0

$$w_0=\frac{w}{(1-v^2/c^2)^{\frac{1}{2}}(1-(v+w)^2/c^2)^{\frac{1}{2}}}, \tag{6}$$

from which

$$w=\frac{w_0(1-v^2/c^2)}{w_0v/c^2+(1+w_0^2/c^2)^{\frac{1}{2}}}. \tag{7}$$

Substituting this in (5), we get for the change in setting of the transported clock

$$\Delta S=\frac{x-vt}{w_0(1-v^2/c^2)^{\frac{1}{2}}}\left[\frac{w_0v}{c^2}+\left(1+\frac{w_0^2}{c^2}\right)^{\frac{1}{2}}-1\right]. \tag{8}$$

Let us now continue the transport of the clock in the Y direction, at a velocity whose Y component is w. In this case the actual velocity of the clock is $(w^2+v^2)^{\frac{1}{2}}$; the distance traversed is $(y^2+v^2T^2)^{\frac{1}{2}}$. We have then that

$$T=\frac{(y^2+v^2T^2)^{\frac{1}{2}}}{(w^2+v^2)^{\frac{1}{2}}}, \tag{9}$$

from which

$$T=\frac{y}{w}. \tag{10}$$

This will be read by the moving clock as

$$T\left(1-\frac{w^2+v^2}{c^2}\right)^{\frac{1}{2}}=\frac{y}{w}\left(1-\frac{w^2+v^2}{c^2}\right)^{\frac{1}{2}} \tag{11}$$

and by a clock stationary on the body as $T(1-v^2/c^2)^{\frac{1}{2}}$, giving as the resultant difference in setting

$$\Delta S=\frac{y}{w}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-\left(1-\frac{w^2+v^2}{c^2}\right)^{\frac{1}{2}}\right]. \tag{12}$$

Again we investigate the relationship between w and w_0, the observable velocity. Noting that in this case we have no length contraction, but only the change of clock frequency, which is as $(1-(w^2+v^2)/c^2)^{\frac{1}{2}}:1$, we have

$$w_0=w\Big/\left(1-\frac{w^2+v^2}{c^2}\right)^{\frac{1}{2}}, \tag{13}$$

from which

$$w=\frac{w_0(1-v^2/c^2)^{\frac{1}{2}}}{(1+w_0^2/c^2)^{\frac{1}{2}}}. \tag{14}$$

Substituting this in the expression for ΔS we get for the change in clock setting

$$\Delta S=\frac{y}{w_0}\left[\left(1+\frac{w_0^2}{c^2}\right)^{\frac{1}{2}}-1\right] \tag{15}$$

and a similar expression for the Z direction.

We are now ready to set down the expression for the observed time of transit of the particle, which in the invariant system is t. This time will be read by the clock at the origin and the transported clock in its final position as $t(1-v^2/c^2)^{\frac{1}{2}}$. From this is to be subtracted the altered setting of the latter clock due to its change of position at the observed velocity w_0. We thus get for the observed time

$$\begin{aligned}t'=&t\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}\\&-\frac{(x-vt)}{w_0(1-v^2/c^2)^{\frac{1}{2}}}\left[\frac{w_0v}{c^2}+\left(1+\frac{w_0^2}{c^2}\right)^{\frac{1}{2}}-1\right]\\&-\frac{y}{w_0}\left[\left(1+\frac{w_0^2}{c^2}\right)^{\frac{1}{2}}-1\right]-\frac{z}{w_0}\left[\left(1+\frac{w_0^2}{c^2}\right)^{\frac{1}{2}}-1\right]\\=&\frac{t-vx/c^2}{(1-v^2/c^2)^{\frac{1}{2}}}-\left[\frac{(x-vt}{c(1-v^2/c^2)^{\frac{1}{2}}}+\frac{y}{c}+\frac{z}{c}\right]\\&\times\left[\frac{(1+w_0^2/c^2)^{\frac{1}{2}}-1}{w_0/c}\right]\\=&\frac{t-vx/c^2}{(1-v^2/c^2)^{\frac{1}{2}}}-\left[\frac{x'+y'+z'}{c}\right]\\&\times\left[\frac{(1+w_0^2/c^2)^{\frac{1}{2}}-1}{w_0/c}\right].\end{aligned} \tag{16}$$

This particular formula reflects the path over which the distant clock was moved. Other formulae of the same general character are obtainable by taking the clock over different paths.

We note at once that if the velocity of transportation of the clock, w_0, is made negligibly small in comparison with the velocity of light, our expressions for x', y', z' and t' degenerate into the Lorentz transformations

$$\begin{aligned}t'&=\frac{t-vx/c^2}{(1-v^2/c^2)^{\frac{1}{2}}},\\x'&=\frac{x-vt}{(1-w^2/c^2)^{\frac{1}{2}}},\\y'&=y,\\z'&=z.\end{aligned} \tag{17}$$

It is important to recognize that these latter expressions are not general nor unconditioned, but are a limiting case, applying only where the velocities of manipulation of our clocks are infinitesimal. They are also the expressions obtained when (distances being measured by rods moved at zero speed with respect to the observing platform) times at distant points are set by light signals *ascribed* the constant velocity c in

all directions in the variant system. In our variant system the velocity of light is a function of c and the observed rod and clock velocities[3] which approaches c as the latter velocities approach zero.

Derivation of the Expression for the "Chronotopic Interval"

A characteristic of the expressions for t', x', y' and z' is that in any actual measurement of a length or a time *the invariant quantities t, x, y and z disappear.* In general *it is impossible, using rods and clocks experiencing the Fitzgerald-Larmor-Lorentz contractions, to determine the elementary factors in the invariant framework.*

The reason for this impasse is mathematically very simple. It arises from the fact that *we have one more unknown quantity than we have equations.* Under these circumstances we can sometimes eliminate one of the unknowns and obtain a useful combination of the others. In the present case eliminating x or t leads to relations in which the variant and invariant quantities cannot be segregated. It is, however, possible by algebraic substitution to eliminate v between t', x', y' and z' and obtain a relation in which the variant and invariant quantities are segregated. This is done by solving for v in the expression for x' and substituting in t'. When this is done, upon assembling the variant and invariant quantities on opposite sides of the equation we obtain the relation

$$c^2t^2-[x^2+y^2+z^2]=c^2t'^2-[x'^2+y'^2+z'^2]+2ct'[x'+y'+z']\left[\frac{(1+w_0^2/c^2)^{\frac{1}{2}}-1}{w_0/c}\right]+[x'+y'+z']^2\left[\frac{(1+w_0^2/c^2)^{\frac{1}{2}}-1}{w_0/c}\right]^2. \quad (18)$$

The left-hand member of this equation is a function of quantities measured in the invariant system, obtainable by physically performable measurements made (right-hand side of the equation) on a moving observation platform, with rods and clocks experiencing the Fitzgerald-Larmor-Lorentz contractions (variant system).

Since the terms involving w_0 depend for their form on the precise path over which the clock is transported, but for any path become zero when $w_0=0$, we may conveniently rewrite (18) as

$$s^2=c^2t^2-[x^2+y^2+z^2]=c^2t'^2-[x'^2+y'^2+z'^2]+F(x', y', z', t', w_0), \quad (19)$$

where $F(x', y', z', w_0)$ is a function which becomes zero when $w_0 \leqq 0$.

Geometrical Meaning of the "Chronotopic Interval"

The left-hand member of (19), or the right-hand member, with $F(x', y', z', t', w_0)$ deleted [corresponding to the use of infinitely slowly moved rods and clocks, or the assignment of the constant value c to the measured velocity of light] so that it has the same form, has been variously termed "the line element," "the interval," "the chronotopic interval," etc., and used as a fundamental expression, from which rod and clock contractions are derived as phenomena of observation. The adjective "chronotopic" has arisen from the concept that t can be considered in a four-dimensional framework as a similar variable to x, y and z, and that s represents a distance in "space-time." In order to bring this about we use instead of s the quantity $\sqrt{-1}\,s=s'$, and identify the velocity of light with the imaginary pure number $\sqrt{-1}$, obtaining

$$s'^2=t^2+x^2+y^2+z^2.$$

Without pursuing further here the justification or the consequences for this procedure,[4] we may, adhering to the postulates of this paper, note that

$$s=(c^2t^2-[x^2-y^2-z^2])^{\frac{1}{2}}$$

is *dimensionally a simple length* in the invariant framework.[5] It might properly be designated the "invariant measurable length." The immediate question is then, what is this length?

We establish the identity of this length at once by reference to the apparatus from which were obtained the experimental results which led to the postulation of the Fitzgerald-Larmor-

[4] "The essence of this postulate . . . the mystic formula $3\cdot10^5$ km $=\sqrt{-1}$ sec.," Minkowski.

[5] This may be lost sight of, but not altered, by a common practice of employing units in which the velocity of light is unity.

Lorentz contractions, that is *the interferometer*. Let L_s be the length of an interferometer arm, stationary in the ether, or undistorted by motion. Then, for motion at velocity v in the direction of the interferometer arm we have, in invariant units, for the time of journey of a light signal out and back to the origin, taking into account the Fitzgerald contraction,

$$t=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{c+v}+\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{c-v}$$

$$=\frac{2L_s}{c(1-v^2/c^2)^{\frac{1}{2}}}.$$

This we may rewrite as

$$(2L_s)^2=c^2t^2-(x^2+y^2+z^2). \tag{20}$$

Because of the null experimental result we obtain an identical equation for all other directions of motion. If we imagine the end mirrors of the interferometer to be located on a spherical shell of diameter S, concentric with the diagonal mirror we have, since $S=2L_s$

$$s^2=c^2t^2-(x^2+y^2+z^2). \tag{21}$$

We therefore have that *the length s is the undistorted diameter of an interferometer, which, because of the Fitzgerald contraction, returns light signals, no matter what their direction, to the origin in the same time, and maintains this behavior for all uniform velocities of the apparatus.* This length, while just expressed in terms of invariant units may, in accordance with (19) be obtained from measurements made with variant rods and clocks. When so measured the same value is obtained whatever the velocity of the observing platform. The expressions in invariant and variant units have the same *form* when and only when the variant measuring rods and clocks are manipulated infinitely slowly.

Formulae Applying in a Gravitational Field

In a preceding paper,[6] I have used the identification of s with an interferometer dimension to study the behavior of an interferometer in a gravitational field, obtaining therefrom a simple derivation of the Schwartzschild "relativity" expression for gravitation. Considerable light is thrown on the interpretation and significance of the "chronotopic interval" by developing it, along the lines of the preceding paragraph, for the case of a gravitational field.

Assumptions used in the paper referred to are (1) that the gravitational field is uniform over the space studied; (2) that a body stationary in the gravitational field suffers a contraction along the direction of the field in the ratio $\sqrt{\gamma}:1$, (where $\gamma=1-2GM/Rc^2$), being unaffected in the direction transverse to the field; (3) that a clock stationary in a gravitational field runs slower in the ratio $\sqrt{\gamma}:1$. From these assumptions it follows at once, for an interferometer to give an unaltered performance when stationary in a gravitational field, that the velocity of light along the field (c_R) must be $c\gamma$, across the field (c_T) it must be $c\sqrt{\gamma}$.

Taking these contractions of length and clock rate, and these velocities of light, we obtain, by following exactly the steps used earlier in this paper, the following expressions for x', y', z' and t', the distances and times of travel of a uniformly moving particle as measured on a platform moving with uniform velocity v, in the same gravitational field:

$$x'=\frac{(x-vt)/\sqrt{\gamma}}{(1-v^2/c^2\gamma^2)^{\frac{1}{2}}},$$

$$y'=y,$$

$$z'=z,$$

$$t'=\frac{(t-vx/c^2\gamma^2)\sqrt{\gamma}}{(1-v^2/c^2\gamma^2)^{\frac{1}{2}}}-\left[\frac{x'+y'+z'}{c}\right]\times\left[\frac{(1+w_0^2/c^2)^{\frac{1}{2}}-1}{w_0/c}\right] \tag{21}$$

and, by eliminating v,

$$s^2=c^2\gamma t^2-y^2-z^2-x^2/\gamma=c^2t'^2-(x'^2+y'^2+z'^2)+F(x',y',z',t',w_0). \tag{22}$$

[Expressed in differentials, for a radial field in a plane in polar coordinates, this, as shown in the preceding paper, is the Schwartzschild formula

$$s^2=c^2\gamma dt^2-r^2d\theta^2-dr^2/\gamma.]$$

[6] H. E. Ives "Behavior of an Interferometer in a Gravitational Field," J. Opt. Soc. Am. **29**, 183 (1939).

Considering first the formulae (22), let us investigate the *measured* velocity of light, taking the simple case of $w_0 \leqq 0$. We have

$$c' = \frac{x'}{t'} = \frac{x - vt}{\gamma(1 - v^2/c^2\gamma^2)}.$$

Putting $x/t = c\gamma$ we get at once $c' = c$; that is, we *measure* the velocity of light as though there were no gravitational field. This result carries over into all the typical phenomena such as *observed* contractions of length and clock rate and the first- and second-order Doppler effects—these are all in terms of c.

Considering the formula (22) further we observe that on the right-hand side (observable quantities) the expression is identical with that obtained for the nongravitational case. We are thus led to generalize as follows:

The formula

$$s^2 = c^2t'^2 - (x'^2 + y'^2 + z'^2) + F(x', y', z', t', w_0)$$

expresses the fact that the dimensions of an interferometer which gives a null result are always given by the same formula in terms of observed measurements. (Conversely the appearance of γ on the right-hand side of the formula[7] is only possible if the interferometer does not give a null result.)

When we consider the expression for s in terms of *invariant* units we find it to be different depending on whether or not the gravitational field is present. It is to be remarked that the assumptions which have been used as to the effect of a gravitational field on length and frequency contractions are not the only ones consistent with a null interferometer behavior. Several possibilities are given in Table I.[8]

From this tabulation it appears that various bendings of a wave front near a gravitating body, and various clock rates in a gravitational field are consistent with null interferometer behavior. The choice between these possibilities can be made by recourse to experiment. This brings up the question, since all optical experiments on bodies moving in gravitational fields would yield identical results to those obtained on bodies in "free space," what experiments are available?

[7] As in the treatment by Phillips, J. Math. and Phys. **1**, 3, 177 (1922).

[8] These and others could be embraced by a general formula of the character developed in H. E. Ives "Graphical Exposition of the Michelson-Morley Experiment," J. Opt. Soc. Am. **27**, 5, 177 (1937).

TABLE I.

	Stationary Interferometer in Gravitational Field	Measured Distances and Times for Interferometer Measured from Moving Platform (rod and clock velocities $\leqq 0$)	Corresponding Formulae for s^2
(1)	$L_T = L_0$ $\quad L_R = L_0\sqrt{\gamma}$ $c_T = c$ $\quad c_R = c\sqrt{\gamma}$ $\nu = \nu_0$	$x' = \dfrac{x - vt}{\sqrt{\gamma}(1 - v^2/c^2\gamma)^{\frac{1}{2}}}$ $y' = y$ $z' = z$ $t' = \dfrac{t - vx/c^2\gamma}{(1 - v^2/c^2\gamma)^{\frac{1}{2}}}$ $c' = c$	$s^2 = c^2t^2 - y^2 - z^2 - x^2/\gamma$ $= c^2t'^2 - y'^2 - z'^2 - x'^2$
(2)	$L_T = L_0\sqrt{\gamma}$ $\quad L_R = L_0\gamma$ $c_T = c\sqrt{\gamma}$ $\quad c_R = c\gamma$ $\nu = \nu_0$	$x' = \dfrac{x - vt}{\gamma(1 - v^2/c^2\gamma^2)^{\frac{1}{2}}}$ $y' = y/\sqrt{\gamma}$ $z' = z/\sqrt{\gamma}$ $t' = \dfrac{t - vx/c^2\gamma^2}{(1 - v^2/c^2\gamma^2)^{\frac{1}{2}}}$ $c' = c$	$s^2 = c^2t^2 - y^2/\gamma - z^2/\gamma - x^2/\gamma^2$ $= c^2t'^2 - y'^2 - z'^2 - x'^2$
(3)	$L_T = L_0$ $\quad L_R = L_0\sqrt{\gamma}$ $c_T = c\sqrt{\gamma}$ $\quad c_R = c\gamma$ $\nu = \nu_0\sqrt{\gamma}$	$x' = \dfrac{x - vt}{\sqrt{\gamma}(1 - v^2/c^2\gamma^2)^{\frac{1}{2}}}$ $y' = y$ $z' = z$ $t' = \dfrac{(t - vx/c^2\gamma^2)\sqrt{\gamma}}{(1 - v^2/c^2\gamma^2)^{\frac{1}{2}}}$ $c' = c$	$s^2 = c^2\gamma t^2 - y^2 - z^2 - x^2/\gamma$ $= c^2t'^2 - y'^2 - z'^2 - x'^2$

The answer is that they must possess equivalent characteristics to those quoted at the beginning of this article in connection with the detection of the ether, namely, experiments in which the cause of the contractions varies over the field of the experiment. In the case of "free space" these are experiments on variation of velocity (rotation). In the case of gravitational fields these are experiments over regions where γ varies. Such is the space near the sun, where the velocity of light over a wave front varies rapidly, detectable by displacement of star images. Similarly, the gravitational field on the earth is so small compared with that on the sun that the difference of atomic clock rates if real becomes detectable.

Probably the point of greatest significance brought out by this study of the "measurable invariant length" in the gravitational field is that the distinguishing characteristics of the formula (22) are only shown in its expression in invariant or "ether" units. The *mimicry* between the variant and invariant expressions which occurs in the nongravitational case, for rods and clocks manipulated infinitely slowly, no longer exists.

A Generalized Formula for the Doppler-Effect

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THE formulas as usually given for the Doppler-effect are based on two assumptions, namely that the velocity of either the source or the observer or both fall in line with the connection of source and observer and that these velocities are constant.

The formula for the change of frequency is $N_2=N_1(c+v_2)/(c-v_1)$ where N_2 is the observed frequency, N_1 the frequency of the source, v_1 the speed with which the source is moving toward the observer, v_2 the speed with which the observer is moving toward the source and c the speed of sound or of light.

It can be easily shown that this formula is only a special case of a generalized formula which neither presupposes constancy of the direction of the velocities nor that their directions should coincide nor that the velocities are constant in regard to time. For the derivation of this formula let a source and an observer move with velocities which might be any kind of a function of time on any kind of a path in space. Then $\varphi_1(t)$, $\varphi_2(t)$, $\varphi_3(t)$ are the x, y, z rectangular coordinates of the source S at the time t and $\psi_1(\tau)$, $\psi_2(\tau)$, $\psi_3(\tau)$ the rectangular corresponding coordinates of the observer P at the time τ. Any effect of the source S travels through space with a velocity c and such an effect taking place at the source at the time t_n will be observed by P at the time τ_n given by the equation

$$\{[\varphi_1(t_n)-\psi_1(\tau_n)]^2+[\varphi_2(t_n)-\psi_2(\tau_n)]^2+[\varphi_3(t_n)-\psi_3(\tau_n)]^2\}^{\frac{1}{2}}=c(\tau_n-t_n). \quad (1)$$

In this general case the frequency observed by P will also be a function of time defined in the following manner. If the frequency of the effect of the source S is N_1, then in the time interval dt the number of signals sent out by S are N_1dt, which will be received by P in a time interval $d\tau$ related to dt by Eq. (1). The frequency of reception will be

$$N_2=N_1\frac{dt}{d\tau}. \quad (2)$$

The differentiation of Eq. (1) furnishes generally $dt/d\tau$ as a function of t and τ and it would be necessary to eliminate τ by aid of Eq. (1) in order to obtain N_2 in dependence of t. But in some instances this is not necessary as will be shown presently. To form $dt/d\tau$ it is simplest as has been pointed to me by Mr. Kunz of the University of Cincinnati to square Eq. (1) and then differentiate, which furnishes

$$\frac{dt}{d\tau}=\frac{\sum_{K=1}^{3}[\varphi_K(t)+\psi_K(\tau)]\psi_K'(\tau)+c^2(\tau-t)}{\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]\varphi_K'(t)+c^2(\tau-t)}. \quad (3)$$

Substituting

$$\left(\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]^2\right)^{\frac{1}{2}} \quad \text{for} \quad c(\tau-t),$$

we finally obtain

$$N_2=N_1\left\{c+\frac{\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]\psi_K'(\tau)}{\left(\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]^2\right)^{\frac{1}{2}}}\right\}\left\{c+\frac{\sum_{K=1}^{3}]\varphi_K(t)-\psi_K(\tau)]\varphi_K'(t)}{\left(\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]^2\right)^{\frac{1}{2}}}\right\}. \quad (4)$$

For the usual Doppler formula

$$\varphi_1(t)=a-v_1t, \quad \varphi_2(t)=0, \quad \varphi_3(t)=0, \quad \varphi_1'(t)=-v_1,$$
$$\psi_1(t)=v_2\tau, \quad \psi_2(\tau)=0, \quad \psi_3(\tau)=0, \quad \psi_1'(\tau)=v_2.$$

Inserting these values in Eq. (4) we have

$$N_2=N_1\frac{c+v_2}{c-v_1}.$$

If the observer is at rest and the source is not

moving on the line joining it with the observer but moving towards him, then the coordinates are

$$\varphi_1(t)=a-v_1t,\quad \varphi_2(t)=d,\quad \varphi_3(t)=0,\quad \varphi_1'(t)=-v_1,$$
$$\psi_1(\tau)=\psi_2(\tau)=\psi_3(\tau)=0$$

$$N_2=N_1\left\{\frac{c}{c-(a-v_1t)v_1/[(a-v_1t)^2+d^2]^{\frac{1}{2}}}\right\}.$$

Putting $$\frac{a-v_1t}{[(a-v_1t)^2+d^2]^{\frac{1}{2}}}=\cos\delta$$

the frequency $N_2=N_1c/(c-v_1\cos\delta)$ which is a constantly changing quantity. It is higher than N_1, for all values of δ from 0 to 90°, for the latter the frequencies are equal and with increasing δ the frequency is constantly decreasing and lower than N_1. But contrary to often met statements there is no sudden finite jump from a frequency higher than N_1, to a frequency lower than N_1, at the instant of minimum distance from the observer.

If the source is at rest, and the observer moving toward the source but again not on the line joining observer and source, the coordinates are

$$\varphi_1(t)=\varphi_2(t)=\varphi_3(t)=0,$$
$$\psi_1(\tau)=v_2\tau-a,\quad \psi_2(\tau)=d,\quad \psi_3(\tau)=0,\quad \psi_1'(\tau)=v_2$$

and

$$N_2=N_1\left\{\frac{c+(v_2\tau-a)v_2/[(v_2\tau-a)^2+d^2]^{\frac{1}{2}}}{c}\right\}.$$

Introducing again

$$\cos\delta=\frac{v_2\tau-a}{[(v_2\tau-a)^2+d^2]^{\frac{1}{2}}}$$

N_2 reduces to $N_1(1+(v_2/c)\cos\delta)$. The change in frequency is qualitatively similar to the first case but numerically there is a difference. Only for $\delta=90°$ again N_2 equals N_1. The cases $\delta=90°$ are characterized by the fact that the velocities are perpendicular to the distance between source and observer.

This also holds true for the general case. To have the frequencies equal, $dt/d\tau$ must be 1. This demands that

$$\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]\psi_K'(\tau)$$
$$=\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]\varphi_K'(t)$$

or

$$\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)][\psi_K'(\tau)-\varphi_K'(t)=0.$$

If we divide this equation by

$$\{\sum_{K=1}^{3}[\varphi_K(t)-\psi_K(\tau)]^2\}^{\frac{1}{2}}\times\{\sum_{K=1}^{3}[\psi_K'(\tau)-\varphi_K'(t)]^2\}^{\frac{1}{2}}$$

it then represents the cosine of an angle formed by two straight lines one representing the distance between source S at the time t and the observer at the time τ, while the other is the resultant of the velocity of the source S at the time t and that of the observer at the time τ. As the left side of the equation equals zero the two lines are perpendicular to each other.—Let us assume that the source S is connected to a free falling body having a distance a from the observer at the time 0. The general equation can then be written in the simplified form

$$a-\frac{g}{2}t^2=c(\tau-t),$$

$$-g\frac{dt}{d\tau}=c\left(1-\frac{dt}{d\tau}\right),$$

$$N_2=N_1\frac{1}{1-gt/c}.$$

If the source is stationary and the observer falling freely, we have under the same initial conditions as above

$$a-\frac{g}{2}\tau^2=c(\tau-t).$$

Differentiating we obtain

$$\frac{dt}{d\tau}=\frac{g\tau}{c}+1.$$

Solving the first equation for τ furnishes

$$\tau = -\frac{c}{g} \pm \left[\left(\frac{c}{g}\right)^2 + \frac{2(a+ct)}{g}\right]^{\frac{1}{2}}.$$

Substituting this value in the equation for $dt/d\tau$ we obtain

$$N_2 = N_1 \frac{g}{c}\left[\left(\frac{c}{g}\right)^2 + \frac{2(a+ct)}{g}\right]^{\frac{1}{2}}.$$

Here N_2 can never become infinite, as no matter how high a speed the observer attains, a finite time must always elapse between reception of successive signals.

Letters to the Editor

The Use of the Hydrogen Lamp in Absorption Spectrophotometry

Concerning this subject Miss Stücklen has recently published a paper in this Journal.[1] In view of the fact that this paper gives a quite erroneous impression of the actual experimental technique in this field it appears necessary to bring forth some corrections and to present some additional data.

The hydrogen lamp has been employed in absorption spectrophotometry for many years and its use has become more and more widespread. The fact that many laboratories are still working with the spark as light source is primarily explained by the relative high cost of a hydrogen lamp.

The statement of Miss Stücklen given on page 38: "While the constancy in operation of a hydrogen discharge over a long period of time is well known, there is a disadvantage in its use in that it is not a point light source and cannot therefore be focused on the slit" should be rectified in two points in the following manner. The constancy of a hydrogen lamp is not sufficient for quantitative absorption measurements if as primary current the ordinary city current is used without special precautions. It has been shown,[2] however, that by employing iron-hydrogen resistance tubes in the primary current the secondary current can be kept constant sufficiently (within 1 percent). It is true that the hydrogen lamp in its ordinary form is not a point light source, but Miss Stücklen seems to have overlooked the fact that a hydrogen lamp with a "point light source" sufficient for quantitative problems in absorption spectrophotometry is available. Such a lamp has been described by Almasy and Kortüm[3] and has been further improved by Almasy.[4]

The arrangement using two light beams simultaneously, as Miss Stücklen has correctly remarked, is expensive and complicated. However it is superfluous if, as mentioned above, care is taken with the constancy of the hydrogen lamp current. On the other hand there are apparently no outstanding advantages in employing the method described by Miss Stücklen. In addition the maximum precision can not be obtained with this method.[5] This is, however, possible by using "central sectors" as described by Pool.[6] A summary published by v. Halban, Kortüm and Szigeti[2] describes a method based upon the above principles. It seems that this publication has also escaped Miss Stücklen. The same is true of a paper published by Hogness[7] in which the absorption curve of potassium chromate has been determined with the aid of a hydrogen lamp and photoelectric cells. These values are very probably more reliable than the older values quoted by Miss Stücklen.

H. v. HALBAN

Physikalisch-Chemisches Institut der Universität Zürich,
Zürich, Switzerland.
April 8, 1939.

[1] H. Stücklen, J. Opt. Soc. Am. **29**, 37 (1939).
[2] H. v. Halban, G. Kortüm and B. Szigeti, Zeits. f. Elektrochem. 42, 628 (1936). For a more detailed description consult the dissertation of B. Szigeti, Zürich, 1938.
[3] F. Almasy and G. Kortüm, Zeits. f. Elektrochem. **42**, 607 (1936).
[4] F. Almasy, Helv. Phys. Acta **10**, 471 (1937). The firm "Quarzhütte Silectra" Uster (Zürich) manufactures these lamps after the data of F. Almasy. Two types are available, one for use with streaming hydrogen, the other filled and sealed with hydrogen.
[5] It would be too lengthy and besides superfluous to mention the errors which cannot be avoided in using the method described by Miss Stücklen. It is well known that the precision of the results is markedly decreased if one does not compare points of equal blackness in close juxtaposition on the photographic plate.
[6] G. M. Pool, Zeits. f. Physik **29**, 311 (1924).
[7] T. R. Hogness, F. P. Zscheile and A. E. Sidwell, J. Phys. Chem. **41**, 379 (1937).

Reply to Communication by Dr. von Halban

There is evidently a misunderstanding as to the purpose of my short paper entitled "A Simple Method for the Determination of Extinction Coefficients Using the Hydrogen Lamp as Light Source." The well-known and very accurate photoelectric method of intensity measurements which has been developed by Dr. von Halban and co-workers was not mentioned because that method meets a completely different need and represents a degree of refinement not approached by the approximate measurements possible by the method described. The failure to cite references to the more complicated hydrogen lamps which serve as "point light sources" was due to the fact that our emphasis was primarily on the simplicity of a method which would utilize the ordinary hydrogen lamp in use in many laboratories and yet give a sufficient accuracy for the rapid measurement of absorption spectra of solutions of organic compounds. Such measurements are of the utmost importance in the preparation of highly purified organic compounds where speed and ease of measurement are factors. The method which was described offers certain advantages over commonly used methods based on comparative spectra with a spark light source and the results of the two methods were found to be in satisfactory agreement. The advantages of a hydrogen discharge tube such as that described by Almasy and Kortüm[1] and by Almasy[2] and the one designed by Dr. Darby and in operation at Columbia University for six years (private communication) when used with a Spekker photometer, a rotating sector or any of the methods for accurate intensity measurement, are beyond question for high precision work. The expense of the equipment together with rather complicated adjustment necessary make it impractical in a laboratory where the spectrograph is used for different types of measurements and the work does not demand so high a degree of accuracy.

The current supplied in our laboratory has been found to be sufficiently constant so that the iron-hydrogen resistance tubes are unnecessary. With an ordinary resistance in the primary circuit, and a secondary current of from 300–400 milliamperes, the fluctuation amounts to less than 1 percent over a period of 45 minutes.

H. STÜCKLEN

Department of Chemistry,
Mount Holyoke College,
South Hadley, Massachusetts,
May 15, 1939.

[1] Almasy and Kortüm, Zeits. f. Elektrochem. **42**, 607 (1936).
[2] Almasy, Helv. Phys. Acta **10**, 471 (1937).

Necrology

George Ellery Hale

GEORGE ELLERY HALE died in Pasadena, California, on February 21, 1938, after an illness with which he had contended courageously for several years. Loved and honored by all who knew him, he will always be remembered for his many remarkable achievements in solar astronomy and for his extraordinary ability as a founder of organizations for scientific research. The Yerkes Observatory of the University of Chicago, the Mount Wilson Observatory of the Carnegie Institution of Washington, the *Astrophysical Journal*, the International Union for Cooperation in Solar Research, the International Astronomical Union, the National Research Council, the Henry E. Huntington Library and Art Gallery, the California Institute of Technology, and the new observatory on Palomar Mountain now under construction, with its 200-inch telescope, are direct results of his ability to organize and to stimulate others.

He realized that a rich harvest was ripe for the observer who would apply high dispersion spectrographs to the study, not merely of sunlight, but to the light from isolated regions of the sun. The large-scale images of the sun produced by the tower telescopes on Mount Wilson and the high dispersion spectra of their underground spectrographs were planned by him for the purpose of reaping some of that harvest.

Of the many results obtained with these instruments one of the most striking, and one thoroughly typical of Hale's methods of research, was the discovery of magnetic fields in sunspots. Always eager to adapt new scientific methods and theories to solar problems, he saw the possibility that magnetic fields might be produced in solar vortices and realized that the peculiar behavior of lines in the spectra of spots might be due to the effects, then just discovered by Zeeman, of a magnetic field on light. Months of careful observation, testing the polarization of magnetically affected lines in laboratory sources and in sunspots, left no doubt that magnetic fields in sunspots were responsible for certain mysterious displacements of their spectral lines.

The invention and development of the spectroheliograph and the spectrohelioscope are outstanding examples of his inventive genius. He not only perfected these specialized instruments, but observed most efficiently with them, probing into the mysteries of the sun and fitting each newly observed fact into a consistent picture of the sun as a whole. Solar vortices always thrilled him, and in the laboratory, with a tank of water on which rested a layer of smoke, he made minature vortices, seeing in them analogies to those on the sun and getting from them suggestions for new solar observations.

Hale's contributions to science were recognized everywhere, and the honors conferred on him were many and great. He was a member of the leading academies and scientific societies of the world including the Optical Society of America of which he was both a charter member and an honorary member. He was elected President of the International Council of Scientific Unions in 1931 and served as a member of the Committee of Intellectual Cooperation of the League of Nations in 1922.

SETH B. NICHOLSON

Francis G. Pease

FRANCIS GLADHEIM PEASE, astronomer and designer of large telescopes and auxiliary astronomical instruments, died after an illness of only a very few days on February 7, 1938.

Dr. Pease was born in Cambridge, Massachusetts, in 1881, received his early education in Highland Park, Illinois, and studied science and engineering at Armour Institute of Technology in Chicago from which he was graduated in 1901 with the degree B.S. His alma mater later conferred on him both the degrees of M.S. and Sc.D. Shortly after his graduation he went to the Yerkes Observatory where for three years he assisted Professor G. W. Ritchey both in astronomical observations and in optical work. In 1904 he went to the Mount Wilson Observatory where he served as instrument designer and astronomer for the rest of his life with the exception of the year 1907–08 which was spent in Evanston, Illinois as superintendent of the "Scientific Shop." Dr. Pease's membership in the Optical Society of America dates from 1922.

Dr. Pease's qualifications for his work at the Mount Wilson Observatory were almost unique. In him were combined a natural gift amounting at times to real genius for mechanical things, a good training in engineering theory and practice and a growing store of experience in actual observation with astronomical instruments. The success of most of the instruments of the Mount Wilson Observatory is no doubt in a considerable measure due to the ingenuity and care given to their design by Dr. Pease.

Since 1928 Dr. Pease devoted one-half of his time to the problems encountered in the design and construction of the 200-inch telescope for the California Institute of Technology. He was a member of the Engineering Committee of this project as well of the subsequent Construction Committee.

His astronomical observations cover a wide field. Beginning shortly after the completion of the 60-inch telescope he made direct photographic observations of nebulae and star fields including clusters, obtaining very excellent photographs which have been and will continue to be of great value for comparison with later photographs of the

same fields and objects. With the 100-inch telescope he obtained photographs of the moon which are still the finest in existence. Much of his time was devoted to spectrographic work with faint objects, especially extragalactic nebulae. Before the advent of the modern high speed lenses and photographic plates results of value required very long exposures, sometimes for 40 to 80 hours.

In 1919 Dr. A. A. Michelson was invited to Mount Wilson to apply his interferometer method to the measurement of stellar diameters, a work in which Dr. Pease immediately joined. The necessary apparatus was designed and also used by him under Dr. Michelson's general direction at first. Later a 50-foot interferometer was designed and built, and with this instrument Pease greatly extended the observations of stellar diameters. Michelson also made two redeterminations of the velocity of light with Dr. Pease's assistance in the design and operation of the apparatus, the last of which was the one-mile long pipeline evacuated to a pressure of about one thousandth of an atmosphere. A repetition of the ether drift experiment was also carried out by Michelson and Pease, using several modifications of the original apparatus of Michelson and Morley.

J. A. ANDERSON

Jakob Kunz

JACOB KUNZ, Professor of Mathematical Physics at the University of Illinois, died at Urbana, Illinois, of heart failure on July 18, 1938. Although his health had been seriously impaired for several years, he kept up his activity both in teaching and in research to within a few weeks of his death.

Kunz was born in Brittnau, Switzerland, on November 3, 1874. He was educated in the public schools and in the Zürich Polytechnikum, from which he received the B.S. degree in 1897 and the Ph.D. in 1902. For five years thereafter he was privatdozent at the Polytechnikum, after which he spent a year at the Cavendish Laboratory in research with J. J. Thomson. He came to the States in 1908, served as instructor in physics at the University of Michigan for a year, and then was appointed assistant professor of mathematical physics at the University of Illinois, where he remained for the rest of his life, becoming successively associate professor and professor. He was elected to membership in the Optical Society of America in 1933.

For years Professor Kunz was responsible for practically the whole repertoire of graduate courses in mathematical physics. He never gave undergraduate courses and it is doubtful whether he could or would have adapted his standards to the immature and almost passive attitude that is all too common among American undergraduates. His teaching was thorough and remarkably clear, and a long series of graduate students are indebted to him for their grounding in classical theoretical physics. However, he never became reconciled to such modern forms of physical theory as relativity and the quantum mechanics, objecting to their substitution of mathematics for visual images and models. As he was himself thoroughly familiar with the mathematical structure of these theories, his objections and criticisms were occasionally very penetrating; and indeed troublesome to any who had a tendency to accept them on faith or authority.

Although his chief interest was in theory, he also carried on, and directed students in, a continuous program of experimental work. He was probably the first to work with photoelectric cells in this country. He developed his specially sensitized alkali photoelectric cells, in quartz, to a degree of sensitivity combined with freedom from dark current that makes them far superior for certain astronomical purposes to any others available, even today. He took part in four eclipse expeditions with the object of measuring, with his cells, the intensity of the light of the sun's corona. His studies of the photoelectric effect in alkali vapors were notable contributions to the subject. While most of his results are now readily understandable in terms of present atomic theories there is one, the rise of photoelectric yield in potassium and caesium at frequencies well above the series limit, which, though amply confirmed by other observers in quite different ways, is still a mystery.

Professor Kunz was always much interested in the work in other laboratories on the campus, and frequently attended seminars in chemistry, mathematics and engineering. He made helpful suggestions about the physical aspects of problems in these fields and on a number of occasions cooperated in investigations with chemists, engineers and biologists. He will be affectionately remembered as a quaint, enthusiastic and friendly figure on the campus and with especial warmth by the group of students whom he befriended and inspired.

F. W. LOOMIS

Wallace Richard Schaub

WALLACE RICHARD SCHAUB [of the staff of the National Bureau of Standards] died in Yonkers, New York, on August 21, 1938. He was born on November 27, 1906, educated in the public schools, and graduated B.S. in E.E. from Cooper Union, New York, in 1929. In June 1930 he joined the staff of the National Bureau of Standards where he was engaged in photometric work. In collaboration with Messrs. Neeland and Laufer he published a paper in the Journal, vol. 28, p. 280, on "Measurement of the Equivalent Luminous Intensity of Rotating Beacons."

Mr. Schaub became an associate member of the Optical Society of America on January 1, 1938, only a few months before his death.

Walter Thomas Spry

WALTER THOMAS SPRY was born February 28, 1895, in Luzerne County, Pennsylvania. His sudden and very unexpected death on December 10, 1938, was a shock to his friends and associates, of whom there are many in the field of color.

His education was won by hard work. Graduation with high honors from Broadway High School in Plymouth, Pennsylvania, won him a first-year scholarship at Lafayette College. Because of his parents' early deaths it was necessary for him to work his way through the remaining years. This he did by working as a surveyor in the coal mines during summer months, and by tutoring during winter months. In 1918 Lafayette College graduated him with a B.S. degree in chemistry. He immediately became connenected with the Chemical Warfare Department of the U. S. Army.

After the war Mr. Spry was employed by the Gilchrest Chemical Company as assistant superintendent of production; from 1920–24 by the Pennsylvania Rubber Company as chemical engineer; from 1924–27 by the Davison Chemical Company as chemical engineer in charge of production, and from 1927 until his death, as business and production manager of the Munsell Color Company, being elected Vice President and Treasurer in 1935.

During his association with the Munsell Company, Mr. Spry contributed much to the expansion of the use of the Munsell Color System in the fields of art, industry, and science. His idea and development of blank charts which enabled students to assemble their own "Book of Color" at a cost within their reach has played a considerable part in the advancement of color knowledge among university students throughout the country. His chief contribution to color-measuring instruments was in connection with the design and construction of a Lac-Chrom-Meter, developed for grading the yellowness of milk.

Mr. Spry became a member of the Optical Society of America in 1935.

As a member of the Colorimetry Committee of the Society, his voice was seldom heard. Nevertheless his interest in all matters of color in which he was concerned was unfailing. His modest and unassuming nature led him to deprecate any contributions that he might make, yet those which he made in time and materials to this society's work, particularly to the work of the subcommittee which is reviewing the spacing of the Munsell system, has been invaluable. No special charts, on special backgrounds, no new sets of materials, were ever too much to ask him to supply, or the hours of observation necessary to provide a complete set of data for the committee's work.

His never-failing good humor and spirit of cooperativeness in furthering the development and diffusion of color knowledge, whether directly concerned with his work or not, are not among the least of the qualities for which he is remembered.

Mr. Spry is survived by his wife, born Dorothy Marie Baker of Phillipsburg, New Jersey, and by four brothers: Frederick S. Spry of Ithaca, New York; William J. Spry and David W. Spry of Wilkesbarre, Pa.; and Harold E. Spry, Grand Rapids, Michigan.

He was a member of the Masonic Fraternity and affiliated with the John H. B. Latrobe Lodge of Baltimore City. He was a trustee of his neighborhood Presbyterian Church.